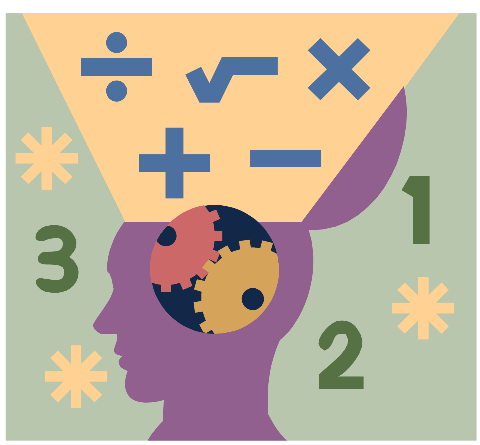
Math & Logic BrainTeasers

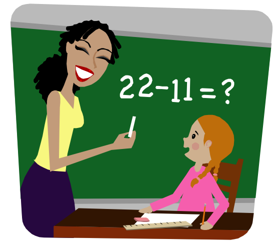
Your listeners may want to have pencil and paper for some of these.



Math

Maximum subtractions

How many times can you subtract 2 from 6?



- Potential Answers
  – Three times: 6 – 2 = 4   4 – 2 = 2   2 – 2 = 0
  – Indefinitely. Continuing from above: 0 – 2 = -2   -2 – 2 = -4   and so on.
- Faulty Assumption
  Assuming subtraction continues until you reach zero (or beyond).
- Correct Answer
  One time. After that you have 4, so you no longer have “6” to subtract from.

Chess games

Two people played 7 games of chess. Each won the same number of games. How is this possible?

- Potential Answer
  One game was a stalemate, so they each won 3.5 games.
- Faulty Assumption
  Assuming they played all 7 games with each other.
- Correct Answer
  They played games with other people as well.

Picking socks

A bag contains 4 pink and 4 blue socks. If you’re blindfolded, what is the fewest number of socks you must remove to guarantee you have 1 of each color?

- Incorrect Answer
  2 socks.
- Faulty Assumption
  Assuming “fewest” is the smallest possible number. While it’s possible to remove 1 of each color in your first two picks, it’s not guaranteed.
- Correct Answer
  5 socks. Potentially, the first 4 socks removed could be all one color. Therefore, a 5^{th} sock is needed to *guarantee* at least one of the opposite color.

Maximum money

A wallet contains $1, $5, and $10 bills. What is the highest dollar value you can withdraw if you have to stop when you get 3 bills of the same denomination?

- Tip (if listeners are struggling to solve this)
  What is the most number of bills you can have of each denomination?
- Correct Answer
  $42. Start with two of each denomination, then maximize value by picking one more of the *highest* denomination: **2**×$1 + **2**×$5 +**3**×$10 = $2+$10+$30 = $42.

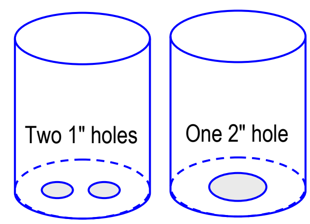
Hotel bill

Three men paid \$7 each for a total of \$21 to share a hotel room. When the manager found they qualified for a special rate, he gave the bellboy \$5 to refund to them. Since the bellboy couldn't figure out how to evenly distribute the money, he gave each man \$1 back and kept the remaining \$2. In effect, each of the three men paid \$6 for a total of \$18. But \$18 plus the bellboy's \$2 adds up to only \$20. What happened to the missing dollar?

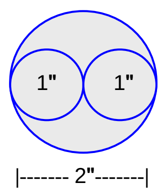
- Faulty Assumption
  Assuming it's correct to treat the men's net loss (–\$18) as a positive number and add it to the bellboy's gain (+\$2) while ignoring the manager's gain and loss.
- Correct Answer
  It's not missing. Of the original \$21, the manager gained (21 – 5=) \$16, the men gained \$3 back, the bellboy gained \$2. Adding gains: \$16 + \$3 + \$2 = \$21.
  Alternate solution: The men lost \$18, the manager lost \$5, the bellboy gained \$2. Adding losses and the gain: –\$18 – \$5 + \$2 = –\$21.

Jar drain

Which drains water faster: A jar with two 1" holes or a jar with one 2" hole?



- Incorrect Answers
  – Two holes empty faster than one.
  – They empty at the same rate since 1" + 1" = 2".
- Correct Answer
  The jar with one 2" hole empties faster.
  – Two 1" holes span but don't cover a 2" hole.
  – A 2" hole has *twice* the πr^2 area:
  Area of two 1" holes = $\pi(½)^2 + \pi(½)^2 = ½\,\pi$
  Area of one 2" hole = $\pi(1^2)$ = $1\,\pi$



Address labels

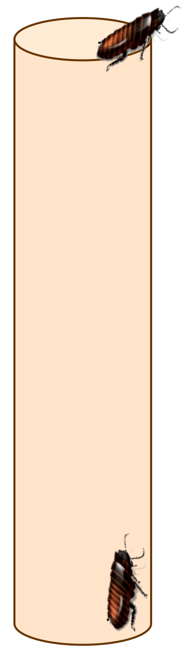
A man has four sealed envelopes to mail, but he accidentally mixes up the address labels. What is the probability that he labels exactly three of the envelopes correctly?

- Incorrect Answer
  3 out of 4 (or 75%)
- Faulty Assumption
  Assuming that the 4th envelope would be mislabeled.
- Correct Answer
  Zero probability. If 3 envelopes are correctly labeled, the 4th must be as well.

Bug crawl

Starting at the bottom of a 15-foot hole, a bug crawls up 3 feet each day but slips down 2 feet each night. When will it emerge from the hole?

- Incorrect Answer
  15 days (Accounts for the 1 ft/day net gain but ignores the max daytime height).
- Faulty Assumption
  Assuming you divide the depth of the hole by the bug's daily net gain.
- Correct Answer
  12½ days (or midway through the 13th day).
  Since the bug nets (3 ft – 2 ft =) 1 ft per day/night period, it will be at 1 ft after the 1st day/night, 2 ft after the 2nd day/night, and 12 ft after the 12th day/night. On the 13th day, the bug starts at 12 ft, crawls up 3 ft during the day, and is out of the 15-ft hole before nightime.



Solution Procedure
First figure out where the bug must start on the final day in order to reach the top of the hole on the up-climb. Then calculate how many day/night periods it took to get to that starting point. Then add ½ day for the daytime up-climb to the top of the hole.

Typical example: 15-foot hole, climbs up 3 feet, slips down 1 foot
- 15 ft depth – 3 ft daytime gain = 12 ft (final day's starting position)
- 12 ft starting position ÷ 2 ft day/night net gain = 6 day/nights (to reach 12 ft)
- 6 + ½ = 6½ days (12+3 up = 15 to get out of hole)

Untypical example: 15-foot hole, climbs up 4 feet, slips down 2 feet
- 15 ft depth – 4 ft daytime gain = 11 ft (final day's *minimum* starting position)
- 11 ft starting position ÷ 2 ft day/night net gain = 5½ day/nights (to reach 10+4 up=14 ft)
  But you must add ½ for the nighttime slippage: 5½ + ½ = 6 (to reach 14–2 down = 12 ft)
- 6 + ½ = 6½ days (12+4 up = 16 to get out of hole)

Tire travel

A bicycle travels 30 miles. Three tires were used equally to complete the trip. How many miles did each tire roll?

- Incorrect Answer
  10 miles each (which would only be true for a unicycle).
- Faulty Assumption
  Assuming that the vehicle-miles traveled equal the tire-miles traveled.
- Correct Answer
  20 miles each.
  If only 2 tires were used, they each would have rolled 30 miles for a total of 60 tire-miles. Since 3 tires were used: 60 tire-miles ÷ 3 tires = 20 miles per tire.

Solution Procedure
Multiply the miles traveled times the standard number of tires on the vehicle.
Divide the tire-miles by the number of tires used.

Example: A car travels 30 miles using five tires equally. How many miles did each tire roll?
- 30 miles × 4 tires = 120 tire-miles
- 120 tire-miles ÷ 5 tires = 24 miles per tire

Logic

Coalminer faces

Two coalminers emerge from a mine at the end of the day and look at each other. One washes his face, the other doesn't. Why not?

- Potential Answers
  – One miner is fastidious; the other isn't.
  – The miner who washes has a date that evening.
- Faulty Assumption
  Assuming the decision to wash is based on something other than seeing each other's faces.
- Correct Answer
  A clean-faced miner sees his partner's dirty face and figures his must be dirty too, so he washes. The dirty-faced miner sees his partner's clean face and figures his must be clean too, so he doesn't wash.

Native lies; visitor truths

In this country, natives always lie and visitors always tell the truth. A man says, "She told me she's a native." Is the man a native or a visitor?

- Tip (if listeners are struggling to solve this)
  Would a native tell someone they were a native?
- Correct Answer
  If she were a native, she would never admit it, because it would mean telling the truth. Therefore, the man lied about what she said, so he must be a native.

Cannibal cookout

You've been captured by cannibals who insist you make a statement. If your statement is True, they will boil you in water. If your statement is False, they will fry you in oil. What statement can you make to prevent them from doing either?

- Tip (if listeners are struggling to solve this)
  Make a statement about the method of cooking that compels the cannibals to do the opposite. But if they do the opposite, they violate their true/false rules.
- Correct Answer
  "You will fry me in oil."
  If they were to fry you in oil that would make your statement True, which means they'd have to boil you in water instead. But if they were to boil you in water that would make your statement False, which means they'd have to fry you in oil. Since this creates a dilemma, they can do neither.

  Alternate answer: "You will *not* boil me in water."

Barber haircuts

A small town has only two barbers. One has a nice, neat head of hair; the other’s hair looks shabby. Which barber should you choose to cut your hair?

- Incorrect Answer
  The neat-headed barber, since he’s more conscientious about his appearance.
- Faulty Assumption
  Assuming a barber’s haircut and skill go together.
- Correct Answer
  The shabby-headed barber. He’s the one who cut the hair of the neat-headed barber and vice versa.

River crossing

Three married couples must get across a river using a boat that holds only two people at a time. The husbands are jealous and won’t allow their wives to be alone with another husband. How many times will the boat have to cross the river to get everyone to the other side?

- Tip (if listeners are struggling to solve this)
  Label the couples Aa, Bb, Cc. Draw a diagram.
- Correct Answer
  Nine crossings. There are several possibilities. The following has each husband row his wife across until all the wives are together, then row each other.

Shore	Row Across	Shore
Aa Bb Cc		---
Bb Cc	Aa $\xrightarrow{1}$	---
Bb Cc	$\xleftarrow{2}$ A	a
A Cc	Bb $\xrightarrow{3}$	a
A Cc	$\xleftarrow{4}$ B	ab
A B	Cc $\xrightarrow{5}$	ab
A B	$\xleftarrow{6}$ C	abc
A	BC $\xrightarrow{7}$	abc
A	$\xleftarrow{8}$ B	ab Cc
---	AB $\xrightarrow{9}$	ab Cc
---		Aa Bb Cc

Make sure *each* row of the diagram has all 6 spouses and that no husband appears alone with others’ wives.