

# Multiples

Multiples are integers formed by multiplying a base factor by a series of factors.

$$\text{Base} \times \text{Series} = \text{Multiples}$$

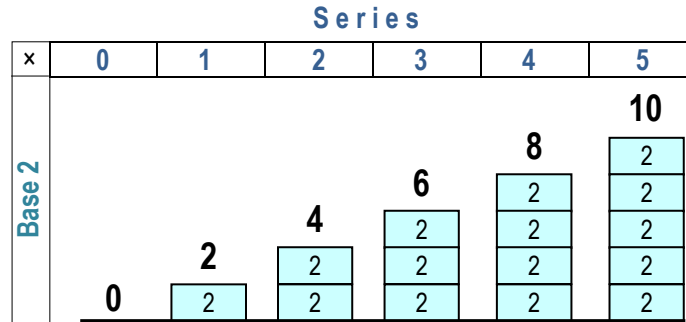
$$\begin{aligned} 2 \times 1 &= 2 \\ 2 \times 2 &= 4 \\ 2 \times 3 &= 6 \end{aligned}$$

Factor × Factor = Product

↓ ↓ ↓

**Base × Series = Multiples**

Base = constant  
Series = 1, 2, 3...

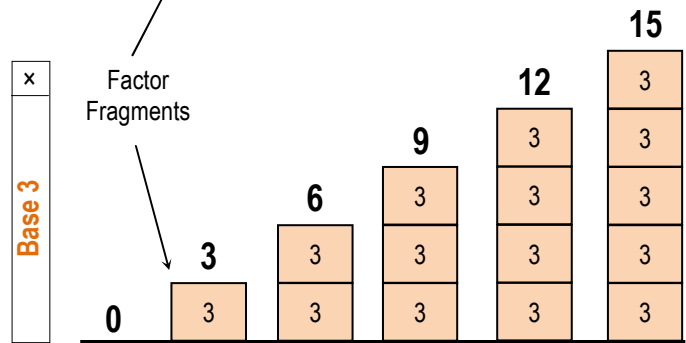


**Zero**  
is a multiple of every Base and 0.  
 $\text{Base} \times 0 = 0$



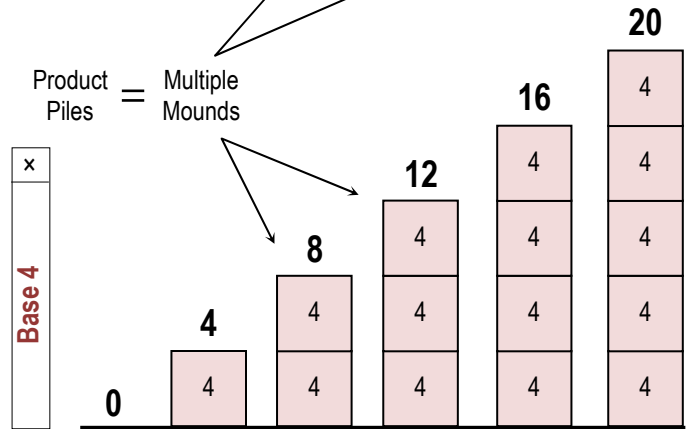
**Multiple Mounds** are Product Piles built from Factor Fragments.

Multiples are **More** than the factors they're built from (providing the factors are > 1).



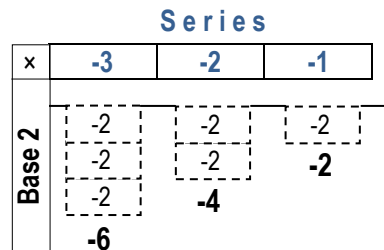
**Every Base** is a multiple of itself and 1.  
 $\text{Base} \times 1 = \text{Base}$

**BaSeM**  
 $\text{Base} \times \text{Series} = \text{Multiple}$   
When building Multiple Mounds, **BaSeM** (base them) on the same base factor.



**All in the Family!**  
If  $M_1$  and  $M_2$  are multiples of a base, then  $M_1 + M_2$  and  $M_1 - M_2$  are also multiples.  
Example: 12 and 8 are multiples of base 4.  
Therefore,  $[12+8=] 20$  and  $[12-8=] 4$  are multiples of 4.

**Can Multiples be Negative?**  
Yes, but the result is negative holes instead of positive piles.



# LCM: Least Common Multiple

The LCM is the *smallest* multiple shared by the given bases.

## LCM from Table

List several multiples for each desired base, then circle the multiples they have in common.

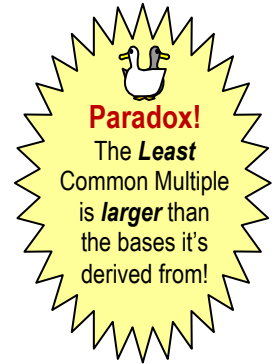
The *smallest* common multiple is the LCM.

		Series				
x	2	3	4	5	6	
Bases	2	4	6	8	10	12
	3	6	9	12	15	18

LCM = 6

Least Common Multiple of bases 2 and 3 is 6.

Common Multiples 6, 12



This holds true for positive LCMs. However, if the larger base is a multiple of the smaller base, the LCM equals the larger base. Example: For bases 2 and 4, the LCM = 4.

## LCMs from Primes



### Load, Crush, Mix, Scoop

Factor each Base product coconut into its prime nutrients.

Load all of the prime factors from the 1<sup>st</sup> coconut into a large cooking pot.

Crush (cross out) any prime factors in the 2<sup>nd</sup> coconut that are already in the pot.

Load what's left.

Mix (magnify/multiply) the factor nutrients into an LCM stew.

Scoop out and multiply each Base by the prime nutrient/s it lacks to make it into the LCM.

8 × 3 = 24  
12 × 2 = 24

4 scoops out (3 × 3) → 4 × 9 = 36  
6 scoops out (2 × 3) → 6 × 6 = 36  
9 scoops out (2 × 2) → 9 × 4 = 36

Multiplying each Base by the product of its scooped out factors yields the LCM.

When you get to the 9, a 3 is already in the pot (from the 6), so you must crush the 9's first 3.

But there isn't another 3 in the pot, so you must load the 9's second 3.

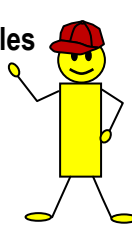
The goal is to have all factors of each Base represented in the pot, *without* duplicating factors.

Why crush? So you don't add excess calories to the stew, which would make the LCM too large and require a Division Diet.

# Why Multiples?

## To Create Times Table

Multiplication Table = Multiples.

	Series											
Bases	2	3	4	5	6	7	8	9	10	11	12	
2	4	6	8	10	12	14	16	18	20	22	24	} Multiples 
3	6	9	12	15	18	21	24	27	30	33	36	
4	8	12	16	20	24	28	32	36	40	44	48	
5	10	15	20	25	30	35	40	45	50	55	60	
6	12	18	24	30	36	42	48	54	60	66	72	
7	14	21	28	35	42	49	56	63	70	77	84	
8	16	24	32	40	48	56	64	72	80	88	96	
9	18	27	36	45	54	63	72	81	90	99	108	
10	20	30	40	50	60	70	80	90	100	110	120	
11	22	33	44	55	66	77	88	99	110	121	132	
12	24	36	48	60	72	84	96	108	120	132	144	

## To Create Equivalent Fractions

Multiply the numerator and denominator by the same Base (Multiply Muscles).

$$\frac{1}{2} \times 2 = \left(\frac{2}{4}\right)$$

Base

$$\frac{1}{2} \times 3 = \left(\frac{3}{6}\right)$$

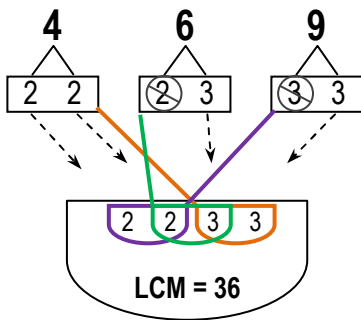
Base

$$\frac{1}{2} \times 4 = \left(\frac{4}{8}\right)$$

Base

## To Find the LCD

When adding or subtracting unlike fractions, the LCM is the LCD (Least Common Denominator).



$$\begin{aligned} 4 \times 9 &= 36 \\ 6 \times 6 &= 36 \\ 9 \times 4 &= 36 \end{aligned}$$

LCM = 36

$$\begin{aligned} \frac{3}{4} + \frac{5}{6} + \frac{7}{9} \\ \frac{3 \times 9}{4 \times 9} + \frac{5 \times 6}{6 \times 6} + \frac{7 \times 4}{9 \times 4} \\ \frac{27}{36} + \frac{30}{36} + \frac{28}{36} \end{aligned}$$

## To Clear Denominators

When an equation has denominators, multiplying by the LCM can make it easier to solve.

$$\frac{z}{6} + \frac{1}{2} = \frac{2}{3} \longrightarrow 6 \left[ \frac{z}{6} + \frac{1}{2} = \frac{2}{3} \right] \longrightarrow z + 3 = 4$$

LCM



# Your Turn!



## *Matching*

- |                         |  |
|-------------------------|--|
| 1) ___ Multiple         | a. Smallest multiple shared by bases.        |
| 2) ___ Base             | b. Multiples shared by bases.                |
| 3) ___ Common multiples | c. Constant on which multiples are built.    |
| 4) ___ LCM              | d. Product formed from Base $\times$ Series. |
| 5) ___ LCD              | e. Smallest multiple shared by denominators. |

## *True or False*

- 6) \_\_\_\_\_ A multiple is an integer.
- 7) \_\_\_\_\_ Zero is a multiple of every base.
- 8) \_\_\_\_\_ Since 16 and 24 are multiples of 8, then  $16+24$  is a multiple of 8.
- 9) \_\_\_\_\_ The Times Table is composed of multiples.
- 10) \_\_\_\_\_ A positive LCM is smaller than the bases it's derived from.

**11) Create a table to find the LCM of 3 and 4.**

**12) Use primes to find the LCM of 15 and 25.**

**13) Find the LCM of 5, 8, and 12.**

**14) Scoop out factors that make 5, 8, 12 = LCM.**

Answers: 1d, 2c, 3b, 4a, 5e, 6T, 7T, 8T, 9T, 10F, 11)12, 12)75, 13)120, 14)24,15,10