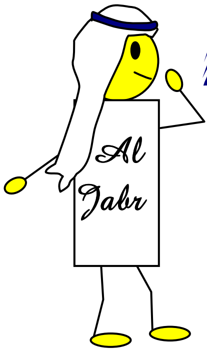
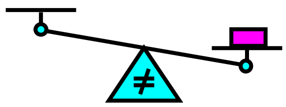
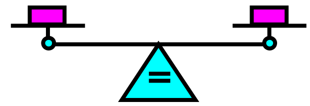


Algebra Antics!





Algebra: Science of Equations

Algebra is the branch of mathematics that uses equations to discover unknown values.
All problems that seek an answer can be thought of as algebra.

1st Grade Algebra
1 + 1 = ?

7th Grade Algebra
1 + 1 = x

Al Jabr
Algebra [AL-jeh-bruh] comes from Al Jabr, which is Arabic for "bringing together." Algebra brings together known and unknown values.

Algebra Anxiety
Algebra's use of simple letters, which seem so friendly in words, creates needless confusion and anxiety in many people.

Algebra Operators

Multiplication

Algebra does *not* use the standard multiplication symbol × because it looks too much like the letter x.

Instead it uses a raised dot
2 • 3
or parentheses
2(3), (2)(3)
or places items together
2a, xy

Division

Algebra does *not* use the standard division sign ÷ or the long division symbol ⟌ .

Instead it uses fraction bars
$\frac{\mathbf{a}}{\mathbf{b}}$
or
a/b

Algebra Terminology

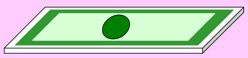
Use the following analogies to compare what you already know, English, with what you are learning, Algebra.

ENGLISH		ALGEBRA	
Word	John	**Term**	1
Phrase	John and Mary	**Expression**	1 + 1
Sentence	John and Mary are together.	**Equation**	1 + 1 = 2

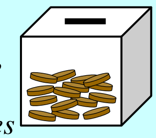
Term: CV^EMD

A term is a mathematical *word* that represents a quantity or value.
As words are built from various letters, terms are built from various components.

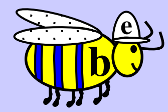
Constants
A term can contain constants [KAWN-stunts], which are numbers that do not vary; e.g., **100** is *always* the number of cents in a dollar.



Variables
A term can contain variables [VAIR-ee-uh-bulz], which are letters that represent numbers that can vary; e.g., **N** is the number of cents in your penny box, which *varies* each time you add or take pennies.



Exponents
A term can contain exponents [EX-poh-nuntz], which are powers assigned to constants or variables, e.g.,



$\mathbf{2^3}$ or $\mathbf{x^4}$

Multiplication
A term can contain several multiplied constants, variables, and exponents; e.g.,

$$3x^2y^4$$

is a single term!

Division
A term can contain several divided constants, variables, and exponents; e.g.,

$$5z^2/2$$

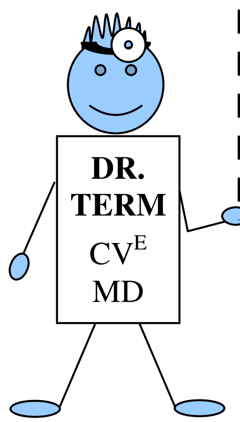
is a single term!

Let Dr. Term help you remember the five items used to build terms, *and two that aren't!*

BrainAid

CV^EMD
CardioVascular Expert,
Medical Doctor

Specializing In
Constants
Variables
Exponents
Multiplication
Division



TRAP!
A term *never* contains addition or subtraction, which are used only to *join* terms.

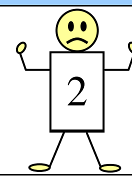
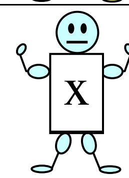
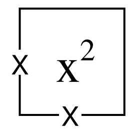
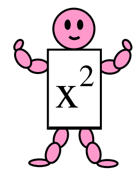
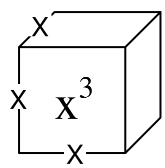
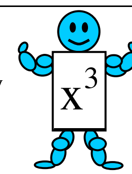
Your turn!
Draw arrows to match the term components to the examples that *best* fit.

Constant	x
Variable	5
Exponent	4y
Multiplication	z/6
Division	x^3

Terms Have Power!

In life, a human family's power is based on how much money or political influence it has. In algebra, a term family's power is based on its exponent.

The higher the exponent, the higher the power.

Power	Family	Visual	BrainAid
x^0 $x^0 = 1$, so x^0 is not written out †	Constant	2	I'm not very strong. 
x^1 $x^1 = x$ so the exponent 1 is not written out	Linear (Line)		I'm sort of strong. 
x^2	Quadratic (Square)		I'm very strong. 
x^3	Cubic (Cube)		I'm extremely strong. 

† Zero Power = 1

You might expect that something raised to the zero power would equal zero, but it actually equals 1, e.g.,

$$1^0 = 1$$
$$20^0 = 1$$
$$500^0 = 1$$
$$x^0 = 1$$

Therefore, constants can omit x^0, e.g.,

$$2x^0 = 2(1) = 2$$

Why it Works

Anything divided by itself equals 1:

$$\frac{x^1}{x^1} = 1$$

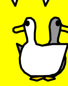
But by the rules of exponents:

$$\frac{x^1}{x^1} = x^{1-1} = x^0$$

Therefore:

$$x^0 = 1$$

(Note: $x \neq 0$)



Paradox!

Since quad means *four*, why is x^2 called quadratic? Because an x by x square has *four* sides.

FYI: x^4 is called "quartic."

Your turn!

Draw arrows to match family and term.

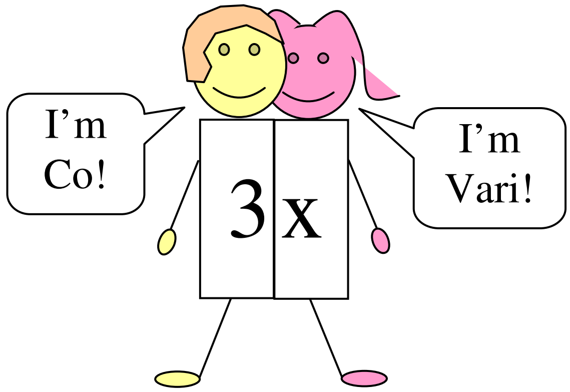
Constant	t^2
Linear	y^3
Quadratic	r^1 or r
Cubic	$4z^0$ or 4

Coefficient: Constant Coworker

Coefficients [coh-ee-FISH-untz] are constants combined with variables.
Coefficients are typically numbers. Coefficients can also be represented by letters.

BrainAid

Vari is able, but her job performance can vary.
Co is constant and helps Vari be more efficient.



Paradox!

Letters normally represent *variables*, but they can also represent *constants.*

To minimize confusion, constant letters are often taken from the beginning of the alphabet (e.g., a, b, c) and combined with variable letters from the end of the alphabet (e.g., x, y, z).

$$(4)x^2 + (3)x + (2)$$

Coefficients

$$(a)x^2 + (b)x + (c)$$

Coefficients without Variables?

2 and **c** appear to be coefficients without variables, but

2 is really $\mathbf{2x^0}$
c is really $\mathbf{cx^0}$.

Since
$\mathbf{x^0 = 1}$

$\mathbf{2x^0 = 2(1) = 2}$
$\mathbf{cx^0 = c(1) = c}$

Therefore, there is no need to write out x^0.

Variables without Coefficients?

If a variable appears without a coefficient, the coefficient is 1, which does not have to be written out, e.g.,

$\mathbf{1x^2 = x^2}$

$\mathbf{1x = x}$

Your turn!

Circle the three coefficients.

$$x^2 + dx + 7$$

What is the coefficient of x^2?

What is the power of x?

What is the variable of 7?

Variable: Box

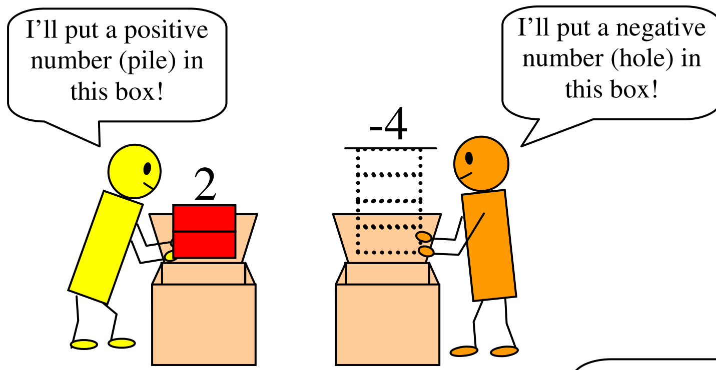
A variable is a letter used as a *placeholder* for a number that can vary.

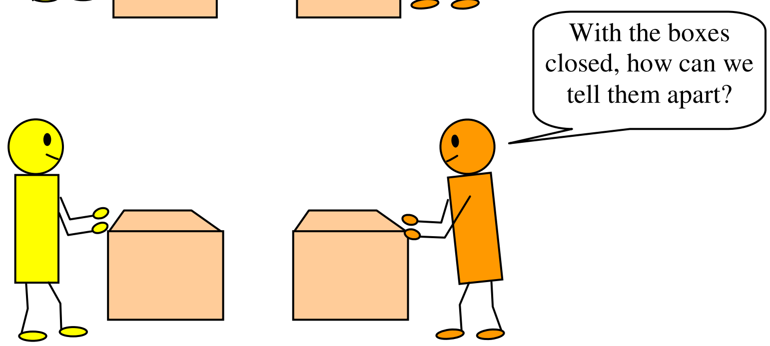
To reduce any anxiety you might have about using letters in math, imagine that a variable is a box.

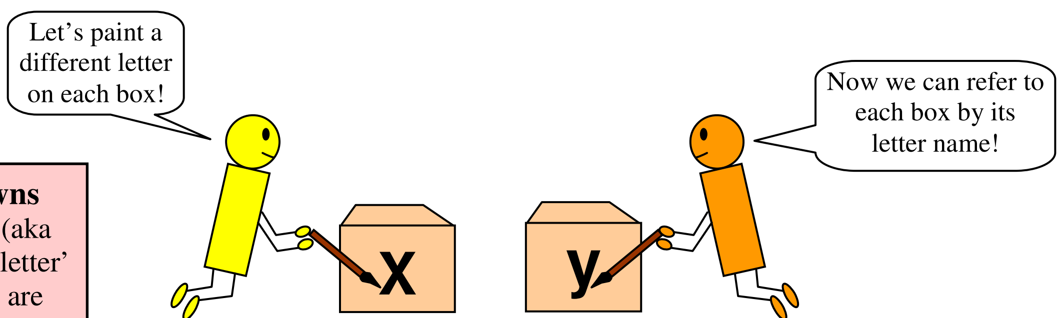
BrainAid

Imagine variables as magic one-size-fits-all boxes that can hold any number, positive or negative, large or small.

Like a genie fitting into a bottle, even a very large number can be put into a box without changing its size.







Unknowns

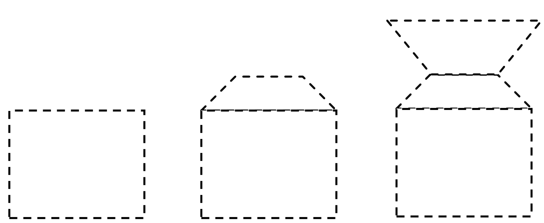
Variables (aka 'literal' or 'letter' numbers) are called *unknowns* when they represent numbers we don't yet know.

Variable letters are equal to whatever is in their boxes.

$$\mathbf{x = 2} \qquad \mathbf{y = -4}$$

Your turn!

Learn to draw boxes by tracing over these.
Start with the face, then the top, then the open lid.



Expression: Poly Mo-Bi-Tri

An expression is a mathematical *phrase* built from a term or terms.

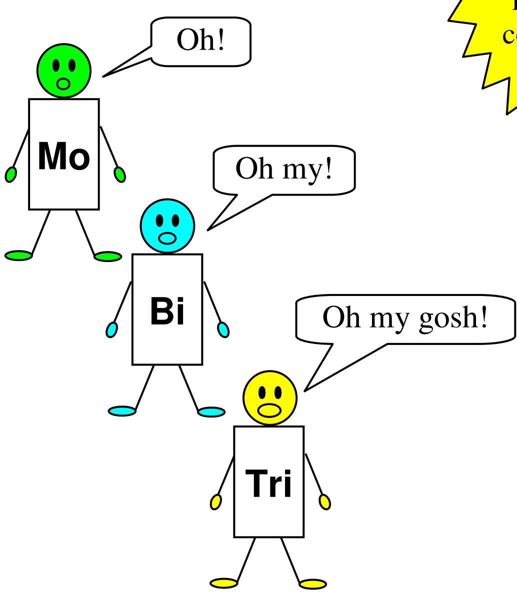
Polynomial Expressions

Polynomial [paw-lee-NOH-mee-ul]
Poly means *many*.
Nomial means *name*, or in this case *term*.
Expressions are classified by how *many terms* they contain.

Type of Expression	# of Terms	Example
Monomial [maw-NOH-mee-ul] (mono means *one)*	**1**	$\mathbf{ax^2}$
Binomial [bii-NOH-mee-ul] (bi means *two*)	**2**	$\mathbf{ax^2 + bx}$
Trinomial [trii-NOH-mee-ul] (tri means *three)*	**3**	$\mathbf{ax^2 + bx + c}$

BrainAid

Imagine three boys, Mo, Bi, and Tri, in a choir singing one, two, and three-word (term) phrases (expressions).



Paradox!

Although poly means *many*, and mono means *one*, monomials are considered to be polynomials.

TRAP!

Polynomial terms, by definition, can have only whole number exponents (e.g., x^2). They can *not* have negative exponents (e.g., x^{-2}) or fractional exponents (e.g., $x^{1/2}$).

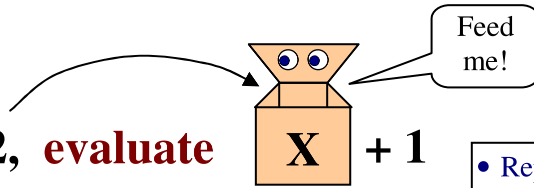
Your turn!

Draw matching arrows.

Monomial	$3v + w$
Binomial	$4u^5$
Trinomial	$z^{-4} + q^{1/3}$
Not Polynomial	$r + s – t$

Evaluating Expressions: Feeding Variables

Evaluate means to substitute (replace) a given value for the variable, then calculate the result.



If $x = 2$, evaluate $x + 1$

$2 + 1$

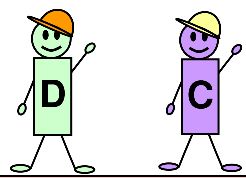
3

- Replace x with 2.
- Calculate result.

BrainAid
E-Valu-Ate
E(xpression) Ate the Value!

Your turn!
Draw an open, hungry box around the variable in the expression. Draw an arrow from the value given in the equation to the box's mouth. Calculate the result.

If $x = 3$, evaluate $2x + 4$



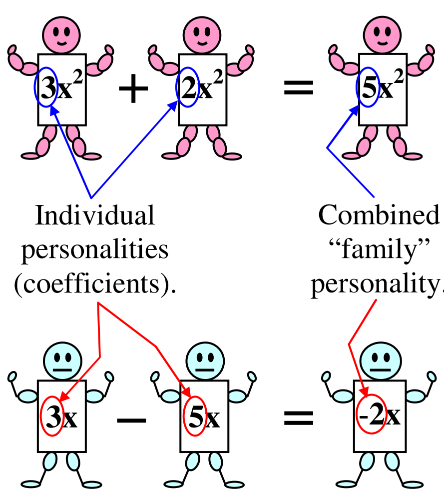
Combining Like Terms

Like Terms
Like (aka *similar*) terms have the *same* variable/s raised to the *same* exponent/s, e.g.,
$2x^3$ and $4x^3$ are like terms.
$3z^2y^4$ and $5z^2y^4$ are like terms.

Combining Like Terms
Like terms can be made into a single term by combining *coefficients*, e.g.,
$$2x^3 + 4x^3 = 6x^3$$
(Think 2 items + 4 items = 6 items)

Why it Works
Distributive Property
Extract common factor x^3:
$2x^3 + 4x^3 = x^3(2+4) = x^3(6)$
Commutative Property
$x^3(6) = 6x^3$

TRAP!
When combining like terms, do *not* change the variables or exponents. It would be like changing the family.



BrainAid
Imagine combining terms' individual positive or negative personalities (coefficients) into a "family" personality.

Your turn!
Draw arrows to match like terms.
Combine coefficients in the center boxes.

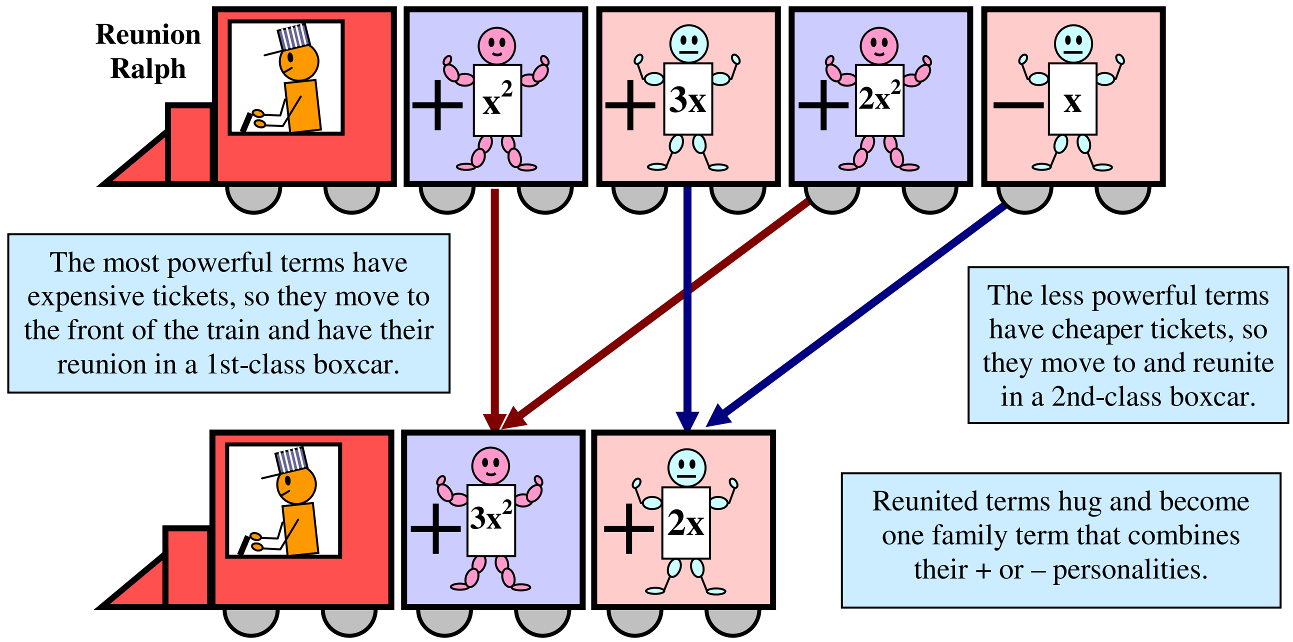
5	☐	← $5x^3y^2$
$3x^3$	☐	← x^2
$-4x^2$	☐	← $-7x^3$
x^3y^2	☐	← 2

Simplifying Expressions: Family Reunion

To simplify an expression, combine like terms.

BrainAid: Imagine terms have boarded a train for their family reunions.

Terms have positive (+) or negative (–) personalities.



Algorithm

- Draw one large rectangle (train) around the entire expression.
- Draw vertical lines to separate each term into its own box (car). Include the + or – sign with each term so you don't overlook it.
- Starting with the highest power terms, draw connecting lines below and to the left for each family. Add or subtract coefficients to combine each family into a single term.

Simplify $3x^2 + 4x - 1 - 2x^2 - x - 3$

$$\boxed{3x^2 + 4x - 1 - 2x^2 - x - 3}$$

$3x^2$	$+\ 4x$	$-\ 1$	$-\ 2x^2$	$-\ x$	$-\ 3$

$x^2 + 3x - 4$

3 terms are simpler to work with than 6!

It's traditional to order terms by power from left to right: $x^3 \quad x^2 \quad x^1 \quad x^0$

TRAP! Do not overlook *minus* signs.

Your turn!

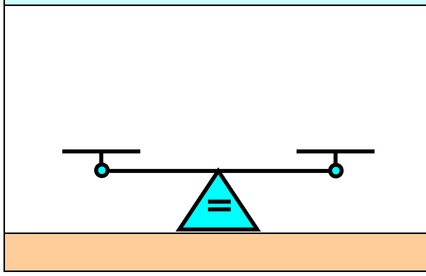
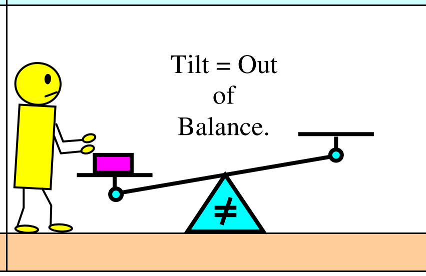
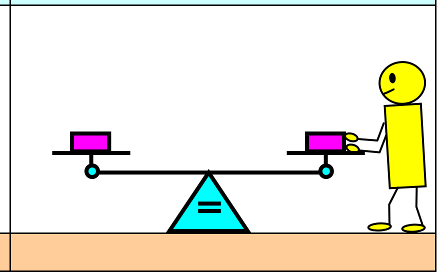
Draw a rectangle around the expression, then add vertical lines to put each term with its + or – sign in a separate box. Simplify by combining like terms (most powerful on the left).

$$3x - 2 + x^2 - 5x - 4x^2 + 2$$

Equation: Balancing Act

An equation is a mathematical *sentence* that equates two expressions.

An equation is like a balance scale that must have equal weight (expressions) on both sides to be balanced.

Start with an empty scale in a balanced condition (indicated by the = sign).	Add a weight to one side to unbalance the scale (indicated by the ≠ sign).	Add an equal weight to the other side to rebalance the scale (indicated by the = sign).
		

Golden Rule of Equations

Whatever you do to one side, do to the other side.

PROPERTY OF EQUALITY

If

$$a = b$$

then

$$a + c = b + c$$

If you add **c** to one side, add **c** to the other side.

$$a - c = b - c$$

If you subtract **c** from one side, subtract **c** from the other side.

$$ac = bc$$

If you multiply one side by **c**, multiply the other side by **c**.

$$a/c = b/c$$

If you divide one side by **c**, divide the other side by **c**.

BrainAid

The Golden Rule of Life says: "Do unto others as you would have them do unto you."

The Golden Rule of Equations says: "Whatever you do to one side, do to the other side."

Your turn!

Draw matching arrows.

Expression	$x = 3$
Equation	$2x + 4$

What is the one thing that an equation has that an expression never will?

BrainAid

Equations have equal signs.

Your turn!

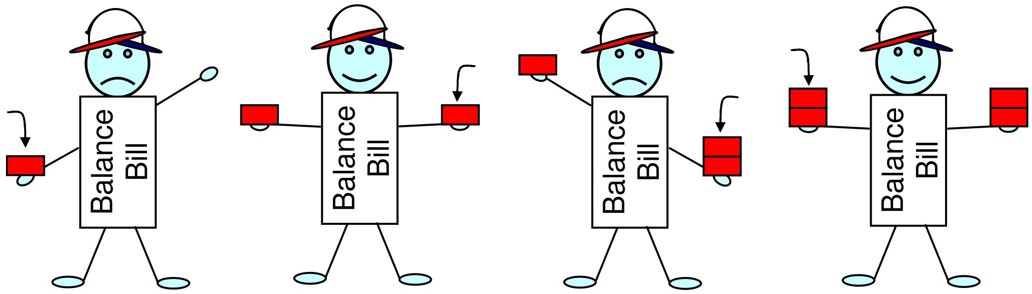
Given **a = b**, apply the Golden Rule by putting numbers in the empty boxes.

$$a + 2 = b + \square \qquad a - \square = b - 3$$

$$a \cdot 4 = b \cdot \square \qquad a / \square = b / 5$$

BrainAid

Balance Bill wears two hats with two bills. Like an equation or scale, he's happy only when his arms are balanced.

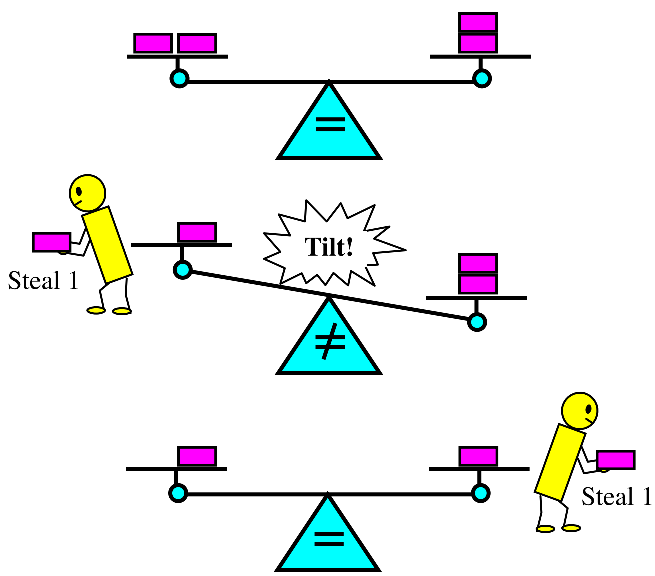


Balancing Equations

Apply the Golden Rule of Equations to keep equations balanced.

Subtracting From Both Sides

If you subtract 1 from the left side, you must subtract 1 from the right side!



Mathematically

$$\not{1} + 1 = 2$$

$$\not{-1}$$

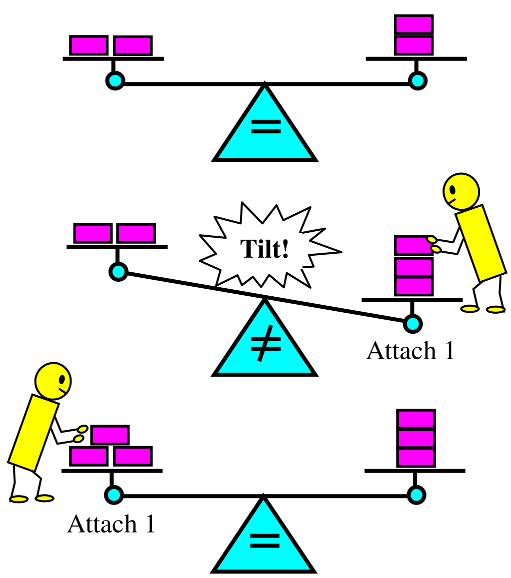
$$1 \neq 2$$

$$-1$$

$$1 = 1$$

Adding to Both Sides

If you add 1 to the right side, you must add 1 to the left side!



Your turn!
Fill in the boxes.

$$1 + 1 = 2$$

$$+\square$$

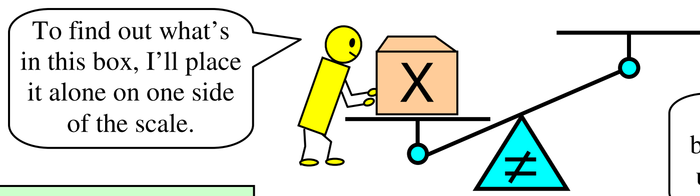
$$2 \neq 3$$

$$+\square$$

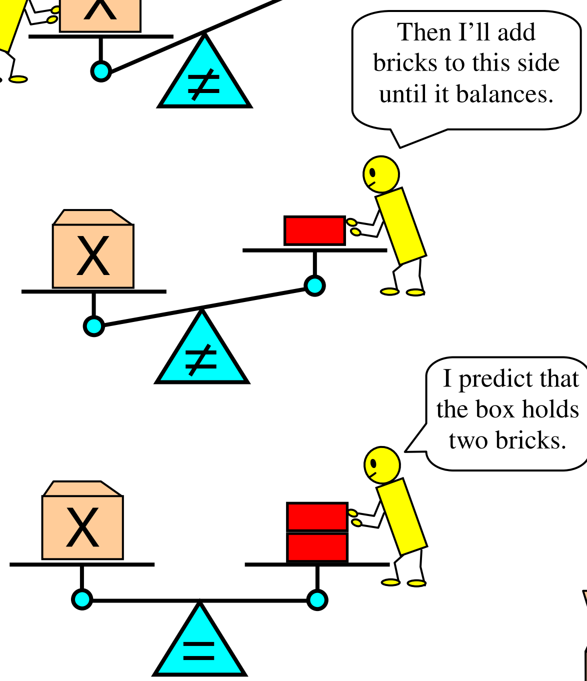
$$3 = 3$$

What’s in the Box?

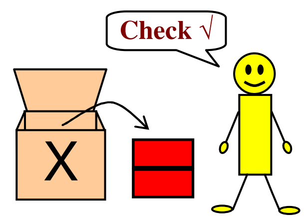
Combining the ideas of a variable being a box and an equation being a scale, the goal of algebra is to figure out what’s inside the box—*without* opening it!



To find the unknown value in a closed box, place it all *alone* on one side of a balance scale. Then add items to the other side until the scale balances. The quantity needed to balance the scale equals the quantity inside the box.



For this to work, the scale has been adjusted to counter the weight of the box; i.e., the box material itself is *weightless* as far as the scale is concerned.

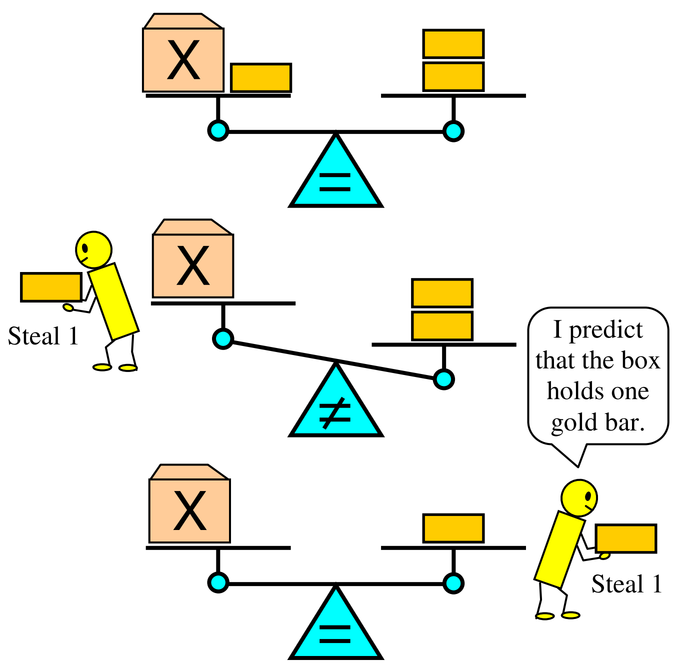


Get the Box Alone!

If the variable box has anything with it on the scale, you must get it alone to determine what’s inside.

If the box has other items with it on the scale, *clear* everything away from the box until it’s alone.

Per the Golden Rule, clear equal amounts from the opposite side to balance the scale.



Your turn!
Fill in the boxes.

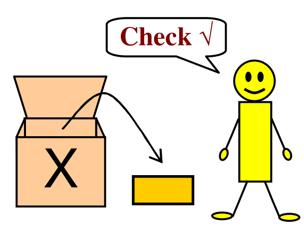
$$x + 1 = 2$$

$$-\square$$

$$x \neq 2$$

$$-\square$$

$$x = 1$$



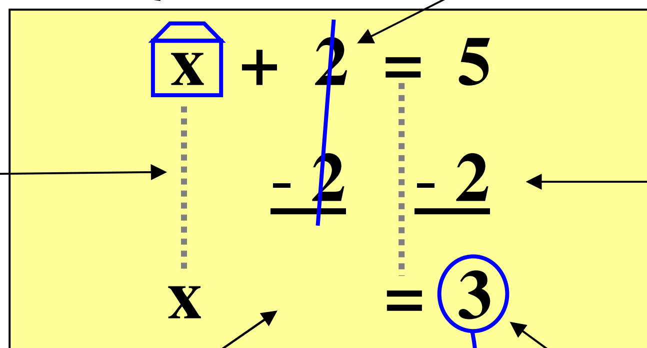
Algebra Arithmetic: Doing the Math

Algebra arithmetic proceeds vertically, line-by-line, down the page. These tips can help.

What's in The Box?
Draw a closed box around the variable. Your goal is to find out what's inside.

$$x + 2 = 5$$

Slash to Cancel
Cancel cleared items with a *solid* slash. There is no need to write the resulting zero beneath.



Rain Items Down
Rewrite variables, equal signs, and numbers straight down, like falling rain.

Golden Rule
Do the same thing to both sides to keep the equation balanced.

Gaps Are Okay
Raining items down may create gaps, but it will keep things organized and prevent you from overlooking an item from above.

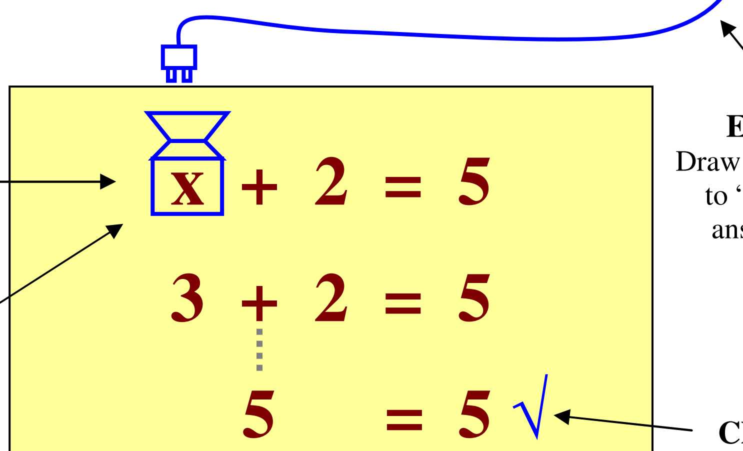
Circle the Answer
Draw a circle around the final answer, so you can easily find it on a page full of items.

Check Your Answer: Plug it in!

Evaluate the *original* equation with your calculated solution. This important step will catch careless arithmetic errors.

Rewrite Original
Always rewrite and evaluate the *original* equation, not a derived version which might contain introduced errors.

Extension Cord
Draw an "evaluation" cord to "plug" your circled answer into the open variable box.



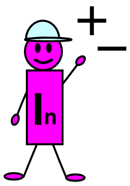
Feed Me!
Draw an open, hungry box around the variable.

Check Mark
If the equation balances, indicate it with a check mark and say "Check!" This can be the most satisfying part!

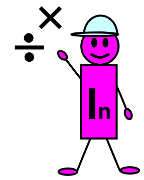
Center Results
Center the result of an operation directly *beneath* the operator that created it.

BrainAid
E-Valu-Ate
E(quation) Ate the Value!

Clearly Opposite!



To get the variable box alone,
perform the *opposite* operation to clear items away from it.



Clearly Opposite uses the **Inverse Property** to cancel or dissolve items.
Additive Inverse: a + -a = 0 Multiplicative Inverse: a(1/a) = 1

x + 1 = 4
If a number is added to the box, subtract it!

$$x + 1 = 4$$
$$\underline{-1 \quad -1}$$
$$x = 3$$

Check:
$$3 + 1 = 4$$
$$4 = 4 \surd$$

Your turn!
Subtract from both sides. Solve and check.

x + 1 = -4 **x + 1 = -4**

x – 1 = 3
If a number is subtracted from the box, add it!

$$x - 1 = 3$$
$$\underline{+1 \quad +1}$$
$$x = 4$$

Check:
$$4 - 1 = 3$$
$$3 = 3 \surd$$

Your turn!
Add to both sides. Solve and check.

x – 1 = -3 **x – 1 = -3**

2x = 6
If a number multiplies the box, divide by it!

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

Check:
$$2(3) = 6$$
$$6 = 6 \surd$$

Your turn!
Divide both sides. Solve and check.

-2 x = -6 **-2 x = -6**

<u>D</u>ivision <u>D</u>ash <u>D</u>issolves
Draw a *dashed* line through items to indicate they dissolve to 1 (instead of cancel to 0).

Use Parentheses to hold a multiplied value.

Eventually, you might want to stop drawing boxes and plugs, but it helps to retrace your work if you draw circles, lines with arrows, and check marks.

$\frac{x}{3} = 2$
If a number divides the box, multiply by it!

$$3\left(\frac{x}{3}\right) = (2)(3)$$
$$x = 6$$

Check:
$$\frac{x}{3} = 2$$
$$\frac{6}{3} = 2$$
$$2 = 2 \surd$$

Your turn!
Multiply both sides. Solve and check.

$\frac{x}{2} = 5$ $\frac{x}{2} = 5$

Clear A/S M/D!

If the box is surrounded by more than one operator (+ – × ÷), it's not always clear what to clear away first. In fact, it's as *clear as mud,* which of course is not clear at all! *Clear A/S M/D* will remind you to clear items away in reverse PEMDAS (aka SADMEP) order, i.e.,

1st: Clear Addition or Subtraction (A/S).

2nd: Clear Multiplication or Division (M/D).

Clear A/S M/D
Clear operators in this order:
Addition
Subtraction
Multiplication
Division

PEMDAS
Check answers in this order:
Parentheses,
Exponentiation
Multiplication or Division
Addition or Subtraction

2x + 1 = 7

Clear Addition, Then Multiplication.

② ①

$2x + 1 = 7$

$-1 \quad -1$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

Multiply, Then Add.

① ②

$2x + 1 = 7$

$2(3) + 1 = 7$

$6 + 1 = 7$

$7 = 7$ √

Your turn!

Clear Addition, then Multiplication.

$3x + 1 = -5$ $\qquad$ $3x + 1 = -5$

x/4 – 2 = 0

Clear Subtraction, Then Division.

② ①

$x/4 - 2 = 0$

$+2 \quad +2$

$4(x/4) = 2(4)$

$x = 8$

Divide, Then Subtract.

① ②

$x/4 - 2 = 0$

$8/4 - 2 = 0$

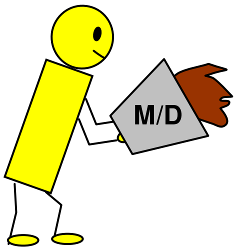
$2 - 2 = 0$

$0 = 0$ √

Your turn!

Clear Subtraction, then Division.

$x/5 - 1 = -9$ $\qquad$ $x/5 - 1 = -9$



Throw M/D!

Sometimes *Clear A/S M/D* is not the easiest way to proceed.
If an equation contains fractions, or coefficients with common factors, it may be simpler to Throw M/D (Multiply/Divide) at it *before* clearing Addition/Subtraction.

Throw M/ to Clear Denominators

A fractionless equation is easier to work with. To clear denominators from an equation, multiply each term by the LCM (Least Common Multiple) of all denominators.

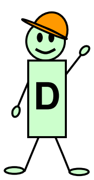
$$\frac{x}{2} - \frac{1}{3} = \frac{1}{6}$$

To eliminate fractions, multiply each term by the LCM of all denominators.

$$6\left(\frac{x}{2} - \frac{1}{3}\right) = \left(\frac{1}{6}\right)(6)$$

$$3x - 2 = 1$$

Dissolve & Distribute
Dissolve each denominator into the LCM first, then distribute the results using the *Distributive Property*.



Your turn!

Multiply to clear denominators.

$$\frac{-3x}{4} + \frac{1}{2} = \frac{1}{3}$$

Throw /D to Reduce Coefficients

Smaller coefficients are easier to work with. To reduce coefficients, divide each term by the GCF (Greatest Common Factor) of all coefficients.

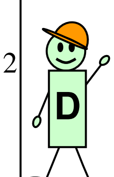
12x + 24 = 36
To reduce coefficients, divide each term by the GCF of all coefficients.

$$\frac{12x + 24}{12} = \frac{36}{12}$$

$$x + 2 = 3$$

Division Distribution
The fraction bar beneath 12x + 24 is equivalent to parentheses, and dividing by 12 is equivalent to multiplying by 1/12, i.e.,

1/12 (12x + 24).



Your turn!

Divide to reduce coefficients.

$$-15x - 18 = 24$$

Taking Sides

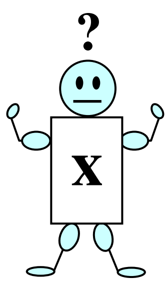
If variables and constants are on opposite sides of the equal sign,
move variables to one side and constants to the other.

$3x + 2 = 4 - 2x$
Move variables to one side and constants to the other side.

$$
\begin{aligned}
3x + 2 &= 4 - 2x \\
\underline{+2x} \qquad &\qquad \underline{+2x} \\
5x + 2 &= 4 \\
\underline{-2} \qquad &\quad \underline{-2} \\
5x &= 2
\end{aligned}
$$

Your turn!
Move variables and constants to opposite sides.

$$7 - 4x = -6x + 5$$

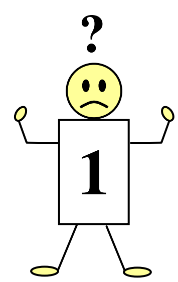


Which Side?

It's traditional to move variables to the left side,
but not mandatory.
For example, **$x = 1$** is the same as **$1 = x$.**

It's generally preferable to move variables to the side
that results in a *positive* coefficient, e.g.,

$$
\begin{aligned}
x + 1 &= 2x \\
\underline{-x} \qquad &\quad \underline{-x} \\
1 &= x
\end{aligned}
$$



Saving Steps

You can save time by moving variables and constants in one step.

$3x + 2 = 4 - 2x$
Move variables and constants in one step!

$$
\begin{aligned}
3x + 2 &= 4 - 2x \\
\underline{+2x} \quad \underline{-2} \quad &\quad \underline{-2} \quad \underline{+2x} \\
5x &= 2
\end{aligned}
$$

Your turn!
Move variables and constants in one step.

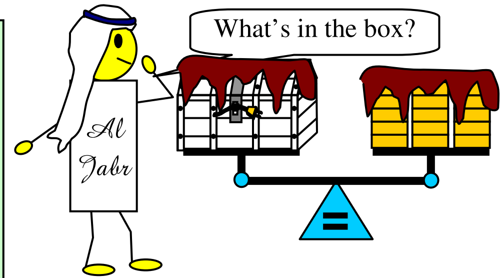
$$7 - 4x = -6x + 5$$

Solving Equations: WAC Golden Plug!

Use these fun steps to solve algebra equations!

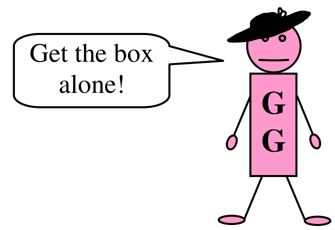
WHAT?

Al Jabr seeks the unknown contents of a treasure box that is covered in mud and sealed with a golden plug. It is balanced on a scale that has gold bars, also covered with mud, on the opposite side. The trick is, Al Jabr must find out what's in the box *without* opening it! In an equation, you'll use the same steps to find the contents of a variable "box."



ALONE!

Not sure what to do, he asks his friend Greta, a former movie star who shunned publicity with her accented lament: I *vant* to be alone! Likewise, she tells Al Jabr that he must *get the box alone!*



CLEAR!

Clearly Opposite!

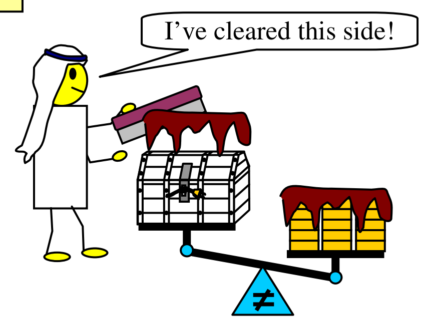
To get the box alone, Al Jabr must clear everything away from it with an opposite (aka inverse) operation. In other words:

- If a number is *added* to the box, he must *subtract* it.
- If a number is *subtracted* from the box, he must *add* it.
- If the box is *multiplied* by a number, he must *divide* by it.
- If the box is *divided* by a number, he must *multiply* by it.

Clear A/S M/D!

If the box is surrounded by more than one operator, it's not always clear what to do first. In fact, it's as *clear as mud*, which of course is not clear at all. Clear A/S M/D means that Al Jabr should proceed in reverse PEMDAS (SADMEP) order, i.e.,

- First clear Addition or Subtraction (A/S).
- Next clear Multiplication or Division (M/D).



Throw M/D!

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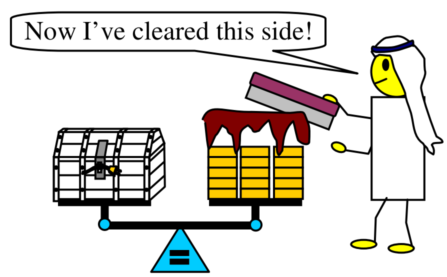

GOLDEN!

To keep his measuring scale balanced, Al Jabr must follow the Golden Rule of Equations: Whatever he clears from one side of the scale, he must clear the same amount from the other side.

PLUG!

Once he thinks he knows what's in the box, Al Jabr takes his sword to "WAC" the golden plug that's holding it shut, then looks inside to confirm his scale findings. In an equation, to check your work, you'll "plug" or substitute the calculated value back into the original equation, "feeding" the variable box, i.e., *evaluate*: equation value ate. Important: When evaluating, be sure to follow PEMDAS order!