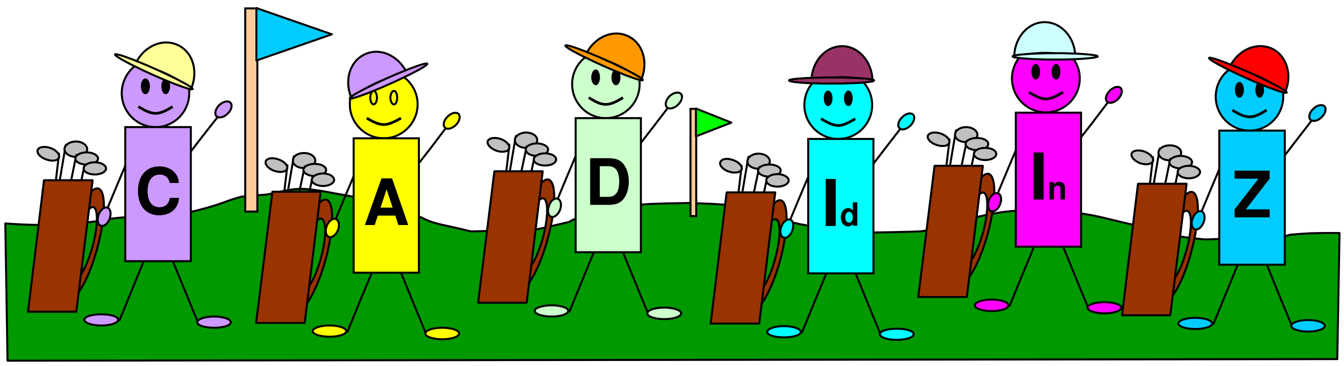
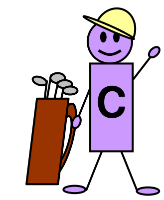
Math Properties: CADIIZ

Math has so many properties, it's tough to remember them all. To learn the six most common ones, imagine 6 caddies (CADIIZ) roaming their respective golf course properties carrying golf clubs for golfers.



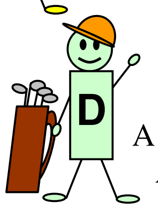


Commutative Property: Change Order

Addends can be attached in any *order*.
Multipliers can be magnified in any *order*.

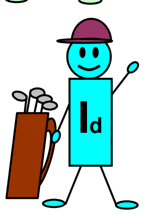
Associative Property: A Social Switch Off

Addends can be attached in any *group*.
Multipliers can be magnified in any *group*.



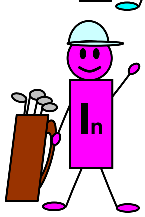
Distributive Property: Rich Uncle

A multiplier magnifies *each* term in a group of terms.
A divisor dissolves *each* term in a group of terms.



Identity Property: Still Me

Any addend plus 0 equals the addend.
Any multiplier times 1 equals the multiplier.

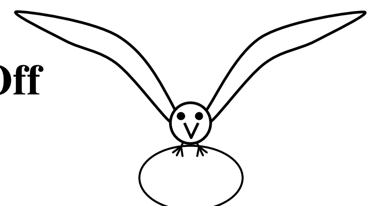


Inverse Property: Opposite Identity

Any addend plus its negative equals 0.
Any multiplier times its reciprocal equals 1.

Zero Property: Black Hole

Any multiplier times 0 equals 0.

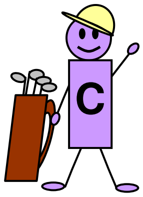


SeeMore the Seagull!

Place numbers on each wing, and slide them into the answer egg.

1 + 2

3



Commutative Property: Change Order

Addends can be attached in any *order*.
Multipliers can be magnified in any *order*.

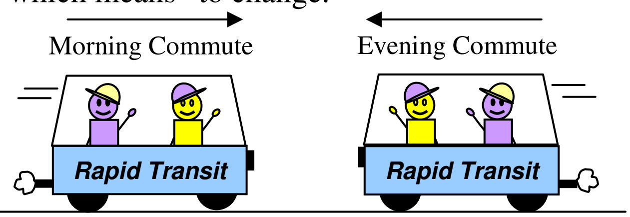
COMMUTATIVE
CHANGE **O**RDER

BrainAid
Imagine CO stands for Change Order.

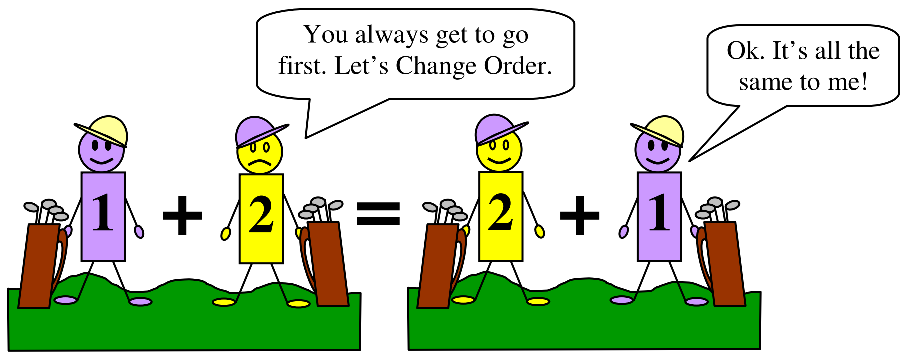
TRAP!
Don't say commuNative. There is no "N" in commuTative.

Commutative [cuh-MYU-tuh-tiv] comes from the word *commute*, which means "to change."

Examples:
The governor *commuted* the prisoner's sentence, changing it from 20 years to only 10.



When caddies *commute* between home and work, they change their direction of travel.



Addition

Changing Order yields the same sum.

$$1 + 2 = 2 + 1$$
$$3 = 3$$

You can commute multiple numbers.

1 + 2 + 3 = 6
1 + 3 + 2 = 6
2 + 1 + 3 = 6
2 + 3 + 1 = 6
3 + 1 + 2 = 6
3 + 2 + 1 = 6

Any order works!

Multiplication

Changing Order yields the same product.

$$1 \times 2 = 2 \times 1$$
$$2 = 2$$

Your turn!

Change Order, then prove the sums are identical.

3 + 4 =

Practical Example

Finding "10" sums

2 + 33 + 8

2 + 8 + 33

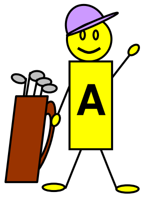
10

43

Your turn!

Change Order, then prove the products are identical.

3 × 4 =



Associative Property: A Social Switch Off

Addends can be attached in any *group*.
Multipliers can be magnified in any *group*.

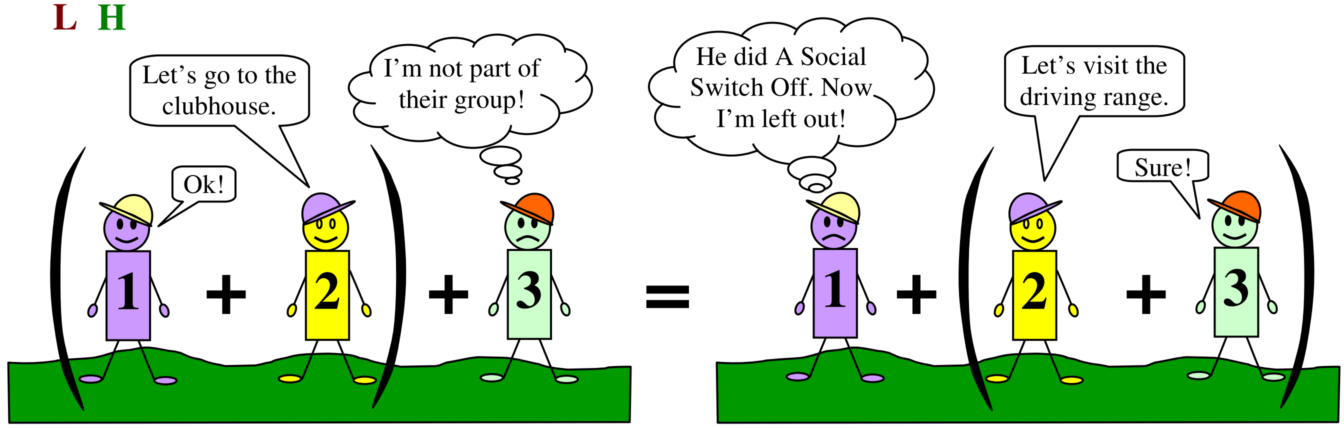
A S S O CIATIVE

- **A**
- **S** OCIAL
- **S** WITCH
- **O** FF

> **BrainAid**
> Imagine ASSO stands for A Social Switch Off.

Associative [uh-SOH-shee-uh-tiv] comes from the word *associate*, which means "to group together."

Examples: Friends like to *associate* with each other. The group of volunteers joined the neighborhood *association*.



Parentheses are used to group items. Operations inside of parentheses are generally performed first.

Addition

A Social Switch Off yields the same sum.

$$(1 + 2) + 3 = 1 + (2 + 3)$$

3 + 3 = 1 + 5

6 = 6

Multiplication

A Social Switch Off yields the same product.

$$(1 \times 2) \times 3 = 1 \times (2 \times 3)$$

2 × 3 = 1 × 6

6 = 6

Practical Example

Finding "10" products

$$(37 \times 5) \times 2$$

$$37 \times (5 \times 2)$$

10

370

Your turn!

Do A Social Switch Off and prove the sums are equal.

$(3 + 4) + 5 =$

Your turn!

Do A Social Switch Off and prove the products are equal.

$(3 \times 4) \times 5 =$

TIP!

Associating does not change number *order:*

1 2 3 ⟷ 1 2 3

But you can commute *and* associate in one problem.

1 2 3 ⟷ 2 1 3

$$(1 + 2) + 3 = 2 + (1 + 3)$$

3 + 3 = 2 + 4

6 = 6

Commutative Property Traps

TRAP!
Subtracted items are too "negative" to commute.

$$1 - 2 \neq 2 - 1$$
$$-1 \neq 1$$

TRAP!
Divided items are too "divisive" to commute.

$$1 \div 2 \neq 2 \div 1$$
$$\tfrac{1}{2} \neq 2$$

Associative Property Traps

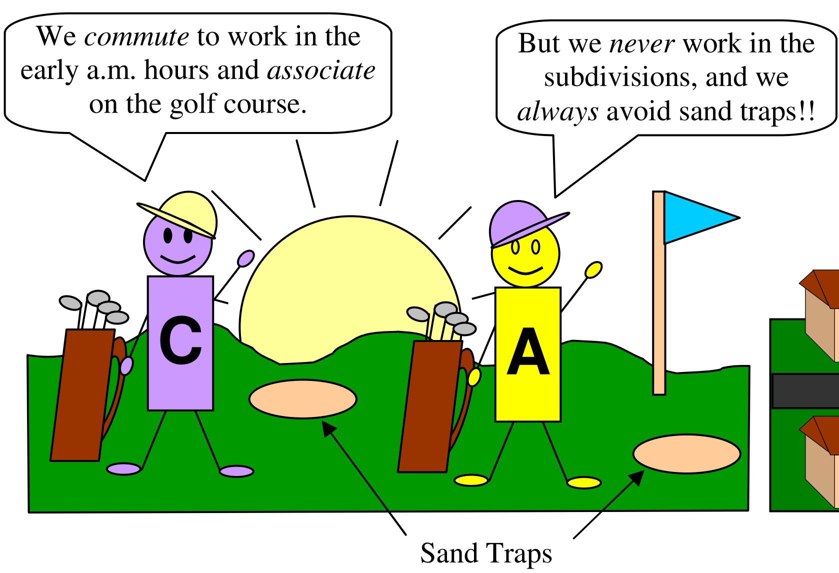
TRAP!
Subtracted numbers are too "negative" to associate.

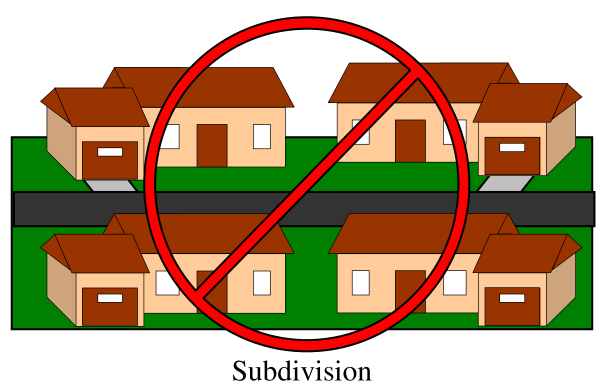
$$(1 - 2) - 3 \neq 1 - (2 - 3)$$
$$-1 - 3 \qquad 1 - (-1)$$
$$-4 \neq 2$$

TRAP!
Divided numbers are too "divisive" to associate.

$$(1 \div 2) \div 3 \neq 1 \div (2 \div 3)$$
$$1/2 \div 3 \qquad 1 \div 2/3$$
$$1/6 \neq 3/2$$

BrainAid
Commutative and Associative caddies work in the a.m. (addition multiplication) on the golf course. They avoid sand traps and do *not* work in the nearby housing subdivision (subtraction division).





Distributive Property: Rich Uncle

A multiplier magnifies *each* term in a group of terms.

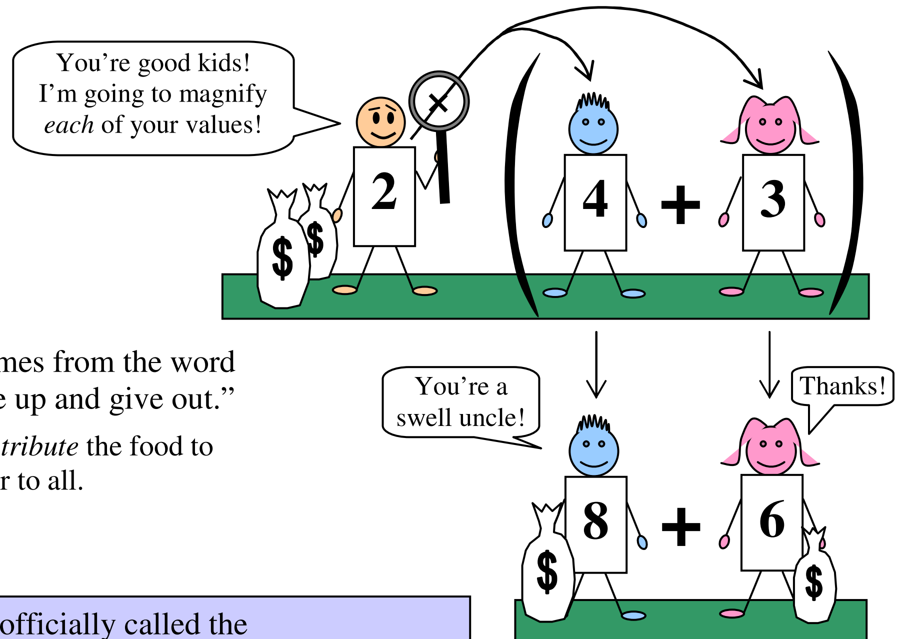
A *term* is a mathematical quantity. The simplest term is a single *number*.	Terms are joined by plus or minus signs, e.g., 4 + 2 – 3 has three terms.	A single term can contain multiplication or division, e.g., 4×3 is one term; 4÷2 is one term.

DISTRIBUTIVE

UNCLE

GIVES

BrainAid
Rich Multiplier Uncle DistribuGives riches.



Distributive [di-STRI-byu-tiv] comes from the word *distribute*, which means "to divide up and give out."

Examples: The charity decided to *distribute* the food to each person. The *distribution* was fair to all.

This property is officially called the
Distributive Property of Multiplication over Addition

Over Addition

A Rich Uncle DistribuGives to *each* grouped term.

$2 \times (4 + 3)$ → $8 + 6$ → 14

PEMDAS Check: $2 \times (4 + 3)$ → 2×7 → 14

Operations inside parentheses are normally performed first, but distributing first yields the same answer.

This property also works *over Subtraction* because subtracting a positive number is equivalent to adding a negative number; e.g., 4 – 3 = 4 + -3.

Over Subtraction

A Rich Uncle DistribuGives to *each* grouped term.

$2 \times (4 - 1)$ → $8 - 2$ → 6

PEMDAS Check: $2 \times (4 - 1)$ → 2×3 → 6

Your turn!

Be a Rich Uncle and magnify the wealth of your niece and nephew! Draw arrows and calculate.

$3 \times (4 + 5)$

Practical Example

Split & Double

2×35

$2 \times (30 + 5)$

$60 + 10$

70

Your turn!

Be a Rich Uncle and magnify the wealth of your niece and nephew! Draw arrows and calculate.

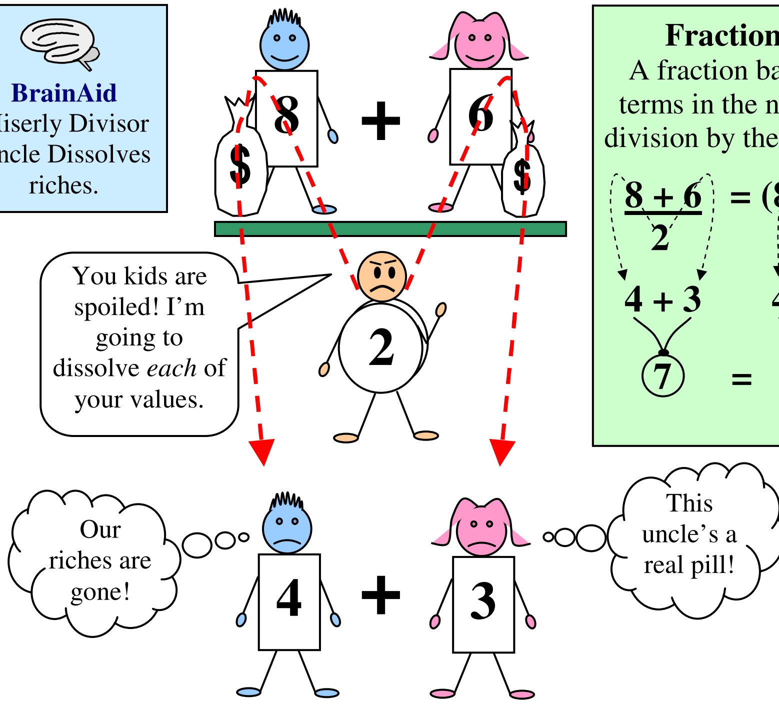
$3 \times (6 - 4)$

Distributive Property: Miserly Uncle

A divisor dissolves *each* term in a group of terms.

This property also works with division because multiplying by a fraction is the same as dividing by its denominator, e.g., ½ × (8 + 6) = (8 + 6) ÷ 2 However, instead of magnifying, the denominator dissolves each term.

BrainAid
Miserly Divisor Uncle Dissolves riches.



Fraction Group
A fraction bar groups all terms in the numerator for division by the denominator.

$$\frac{8+6}{2} = (8+6) \div 2$$

4 + 3   4 + 3

(7) = (7)

Paradox!
Dividing by a number (2) is the same as *multiplying* by 1 over that number (1/2).

½ × (8 + 6)

4 + 3

(7)

TIP! Use *dashed* arrows to indicate dissolving.

Your turn!
Play Miserly Uncle and dissolve the wealth of your niece and nephew. Draw *dashed* dissolving arrows and calculate the answer.

$$\frac{9+3}{3}$$

Practical Example
Split & Halve

76 ÷ 2

(70 + 6) ÷ 2

35 + 3

(38)

Your turn!
Play Miserly Uncle and dissolve the wealth of your niece and nephew. Draw *dashed* dissolving arrows and calculate the answer.

$$\frac{9-3}{3}$$

Distributive Traps

Since the traps on this page don't involve addition or subtraction, they are not technically part of the Distributive Property of Multiplication Over Addition, but they are distributive in nature.

TRAP!

With a term containing multiplication e.g., (4×3), magnify only *one* of the numbers in the term.

Wrong

$2 \times (4 \times 3)$

8×6

~~48~~

Right

$2 \times (4 \times 3)$

8×3

24

Your turn!

Magnify only the first number.

$3 \times (4 \times 5)$

Magnify only the second number.

$3 \times (4 \times 5)$

TRAP!

With a term containing division e.g., (4÷2), magnify only the *first* number in the term.

Wrong

$3 \times (4 \div 2)$

$4 \div 6$

~~4/6~~

Right

$3 \times (4 \div 2)$

$12 \div 2$

6

Your turn!

Magnify only the first number.

$3 \times (6 \div 2)$

Distribution Over Multiplied Numbers

*** Any One ***

For a single term containing multiplied numbers, you can distribute to any one of those numbers.

Why any number? Because multiplication is commutative.

$2\times(49\times5) = 2\times(5\times49)$

Parentheses First?

You could avoid some of these traps by using PEMDAS to evaluate items in parentheses first.

But if parentheses hold variables, use these techniques, e.g.,

$2 \bullet (a \bullet b) \neq 2a \bullet 2b$

$2 \bullet (a \bullet b) = 2a \bullet b$

Distribution Over Divided Numbers

*** First Only ***

For a single term containing divided numbers, you must distribute only to the first number.

Why only the first? Because division is *not* commutative.

$3\times(6\div2) \neq 3\times(2\div6)$

TRAP!

With a term containing multiplication e.g., (4×6), dissolve only *one* of the numbers of the term.

Wrong

$\frac{4 \times 6}{2}$

2×3

~~6~~

Right

$\frac{4 \times 6}{2}$

2×6

12

Your turn!

Dissolve only the first number.

$\frac{9 \times 6}{3}$

Dissolve only the second number.

$\frac{9 \times 6}{3}$

TRAP!

With a term containing division e.g., (8÷4), dissolve only the *first* number of the term.

Wrong

$\frac{8 \div 4}{2}$

$8 \div 2$

~~4~~

Right

$\frac{8 \div 4}{2}$

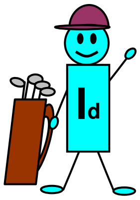
$4 \div 4$

1

Your turn!

Dissolve only the first number.

$\frac{9 \div 3}{3}$



Identity Property: Still Me

Any addend plus 0 equals the addend.
Any multiplier times 1 equals the multiplier.

Numbers combined with Identity Elements (0, 1) remain the same.

S
T M
I D E N T I T Y
L
L

BrainAid
A number retains its identity and says "I'm still me!"

Identity [ii-DEN-ti-tee] refers to the unchanging traits that make something or someone unique.

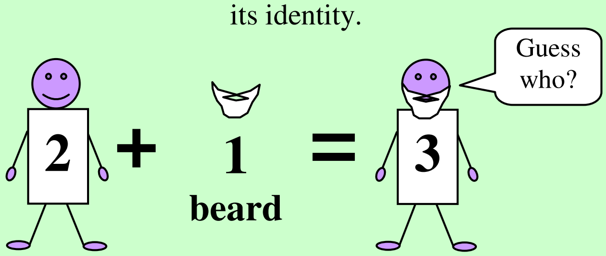
Examples: Scientists discovered the *identity* of the virus. The witness revealed the *identity* of the thief.

Additive Identity Element
Zero Influence

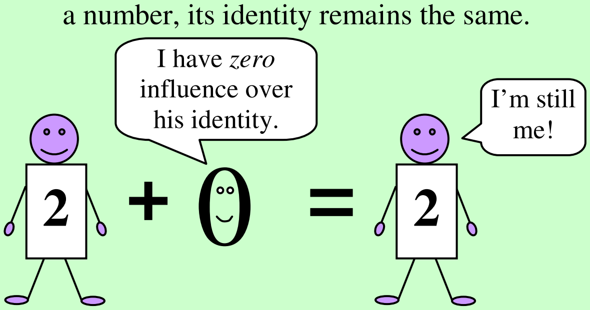
Addend + 0 = Addend

The number 0 is the *Additive Identity Element*. An element is a "thing." Zero is the only "thing" that won't change the identity of the number you add it to.

Adding something, like a beard, to your face changes your identity. Adding something to a number changes its identity.



But if you add nothing (zero) to your face, your identity remains the same. If you add nothing (zero) to a number, its identity remains the same.

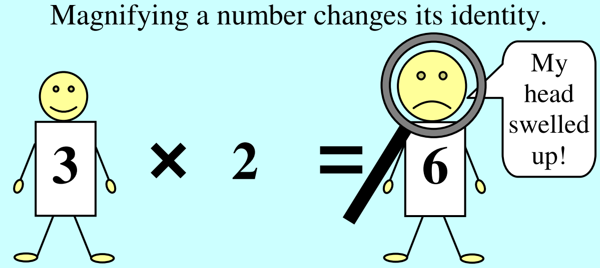


Multiplicative Identity Element
One and the Same

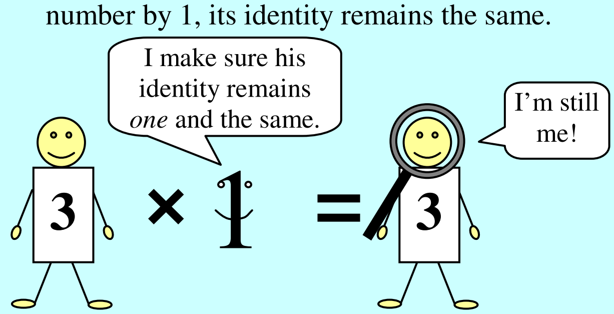
Multiplier × 1 = Multiplier

The number 1 is the *Multiplicative* [mul-ti-PLIK-uh-tiv] *Identity Element*. An element is a "thing." One is the only "thing" that won't change the identity of the number you multiply it by.

Magnifying your face changes your identity. Magnifying a number changes its identity.



But if you magnify your face by 1, it doesn't swell, and your identity remains the same. If you magnify a number by 1, its identity remains the same.



Your turn!

Insert the Additive Identity Element.

3 + ☐ = 3 ☐ + 3 = 3

Practical Use
The concept of multiplicative identity is used to create Equivalent Fractions.

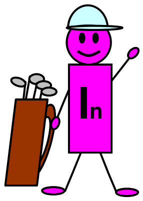
Your turn!

Insert the Multiplicative Identity Element.

4 × ☐ = 4 ☐ × 4 = 4

TRAP! Because subtraction is *not* commutative, 0 is *not* an identity element for subtraction; e.g., 2 – 0 = 2 but 0 – 2 = –2.

TRAP! Because division is *not* commutative, 1 is *not* an identity element for division; e.g., 2 ÷ 1 = 2 but 1 ÷ 2 = 1/2.



Inverse Property: Opposite Identity

Any addend plus its negative equals 0.
Any multiplier times its reciprocal equals 1.

Opposite numbers produce *Identity* Elements: 0 or 1.

OPPOSITE
INVERSE
IDENTITY

> **BrainAid** The Inverse Property is the *opposite* of the *Identity* Property.

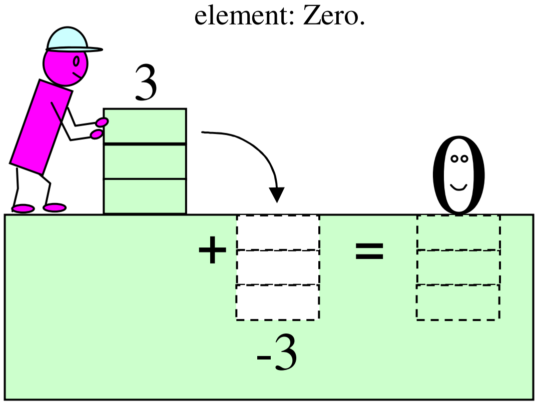
Inverse [IN-vurss] means "opposite." Invert means "to turn or flip over."

Examples: Turning the lid clockwise didn't open the jar; so the caddie turned it in the *inverse* direction. He then *inverted* the jar, pouring out the ketchup.

Additive Inverse
Pile fills Hole

Addend + -Addend = 0

The inverse (opposite) of a positive pile is a negative hole of the same size. Pushing the pile into the hole fills it up, and the sum is the additive identity element: Zero.



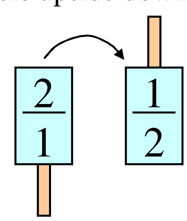
Multiplicative Inverse
Flipping & Melting Popsicles

Multiplier × Reciprocal = 1

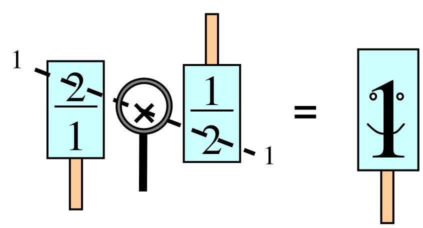
The inverse (opposite) of a multiplier is its reciprocal [ree-SI-proh-kul], which is the multiplier flipped over. A number multiplied by its reciprocal "melts" into the multiplicative identity element: One.

Convert integers to fractions before inverting, e.g., convert 2 to 2/1.

BrainAid
Imagine flipping a popsicle upside down.
Think popsicle.
Think re*flip*rocal.
Think reciprocal.



Magnifying opposite popsicles heats and melts them into one.



Your turn!

Insert Additive Inverses.

$4 + \square = 0 \qquad \square + -5 = 0$

Practical Use
In algebra, inverses are used to clear numbers away from variables.

TRAP!
Reverse + or – with additive inverses, but *not* with multiplicative inverses. Why not? It takes two positive or two negative multipliers to create a positive One.

Your turn!

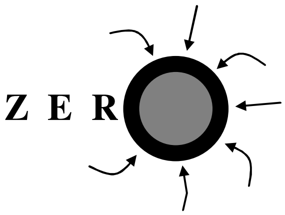
Insert Multiplicative Inverses.

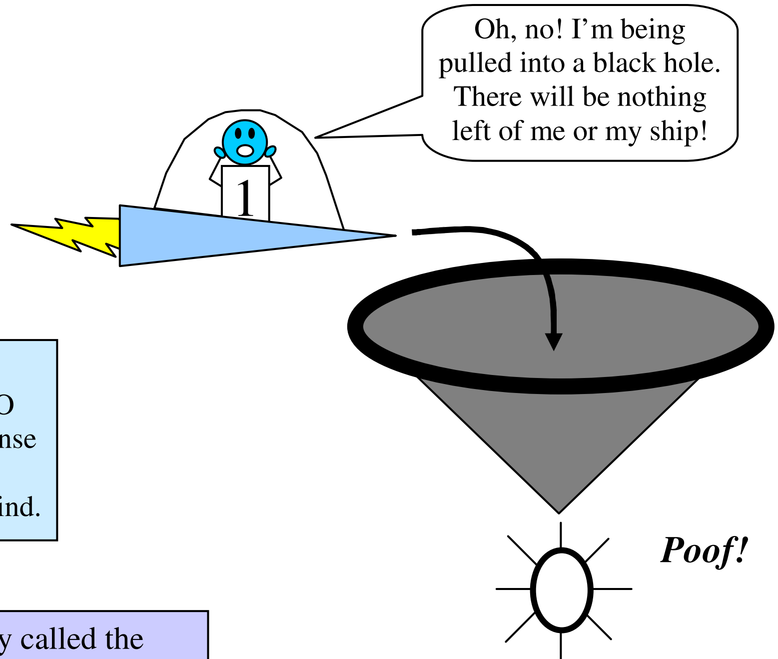
$$\frac{3}{4} \times \square = 1$$

$$-\frac{3}{4} \times \square = 1$$

Zero Property: Black Hole

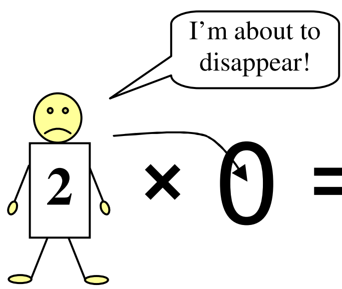
Any multiplier times 0 equals 0.





BrainAid
Imagine that the O in zerO is a black hole in space whose intense gravitation pulls in anything that comes near it, leaving nothing behind.

This property is officially called the
Multiplicative Property of Zero



$\times\ 0 = 0$

Your turn!

Show what emerges from a black hole.

$3 \times 0 = \square$ $\quad$ $-3 \times 0 = \square$

Practical Use
In algebra, the Zero Property is used to find solutions to factored quadratic equations.