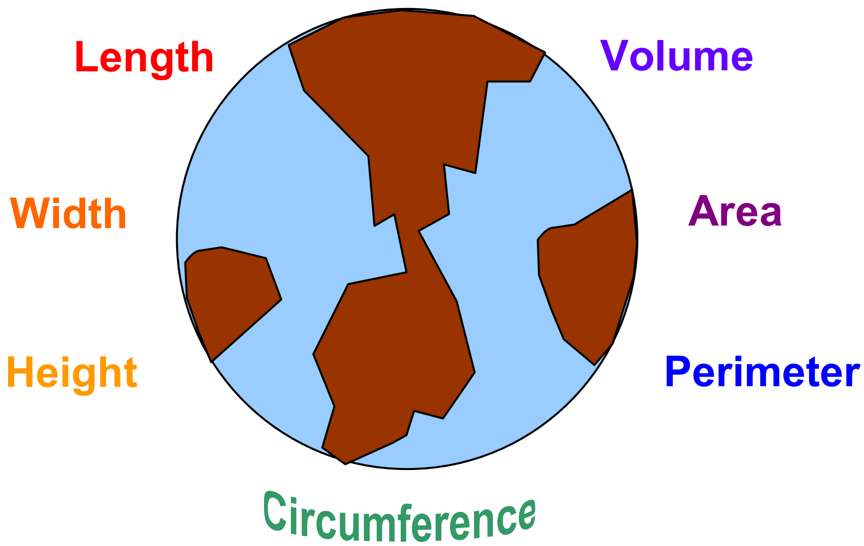
GEOMETRY GEMS

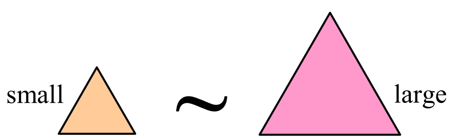
Geo = Earth
Metry = Measure



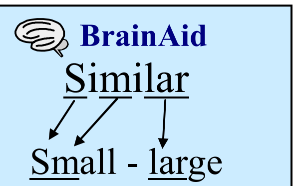
Shape Operators

Geometry uses special operators to show relationships between shapes.

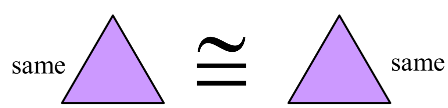
Similar
Same shape, size can vary.



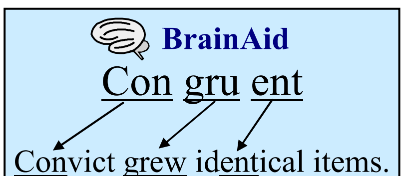
The Similar operator looks like an S lying on its back.

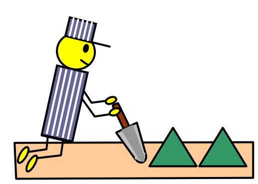


Congruent
Same shape, same size.

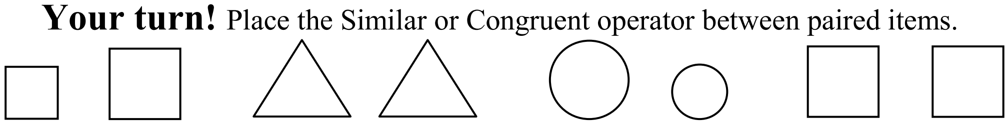


The Congruent [kawn-GRU-ent] operator combines Similar and Equal signs.





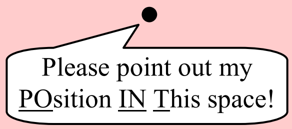
Your turn! Place the Similar or Congruent operator between paired items.



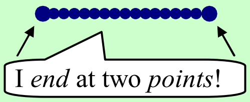
GeoParts

These "Geometry Parts" can be used to build almost any shape.

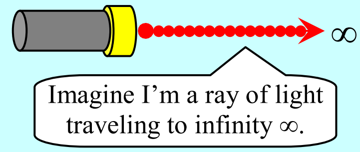
Point
A position in space.



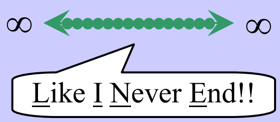
Line Segment
Straight series of points with two endpoints.



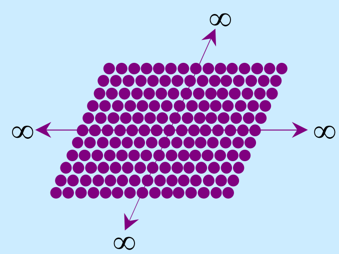
Ray (aka half-line)
Straight series of points with one endpoint. Continues forever in *one* direction.



Line
Straight series of points with no endpoints. Continues forever in *both* directions.

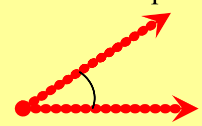


Plane
A flat "panel" of points which continues forever in *all* directions.



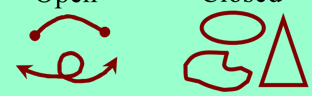
BrainAid: The L in paneL flies up to make a pLane.

Angle
Formed by two rays with a common endpoint.



Curve
A bent segment or line.

Open Closed



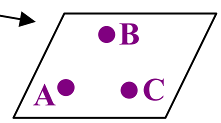
Naming GeoParts

Line Segment $\overline{AB}$

Point A

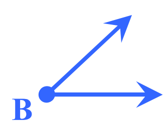
Use UPPERCASE letters to name points, then use those points to name other parts.

Plane ABC



Ray $\overrightarrow{AB}$

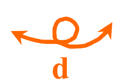
Angle ∠B



Use a lowercase letter to name the *entire* GeoPart.

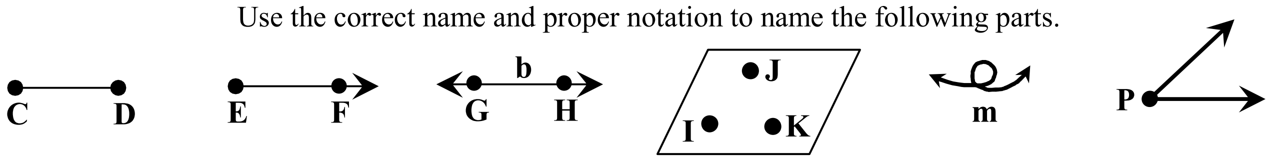
Line $\overleftrightarrow{AB}$ or Line c

Curve d



Your turn!

Use the correct name and proper notation to name the following parts.



Point to (x,y,z) and Me!

Points can be designated by coordinates [coh-OR-di-nutz] on axes [AX-eez].
Each individual axis [AX-iss] is named with a letter: x, y, or z.

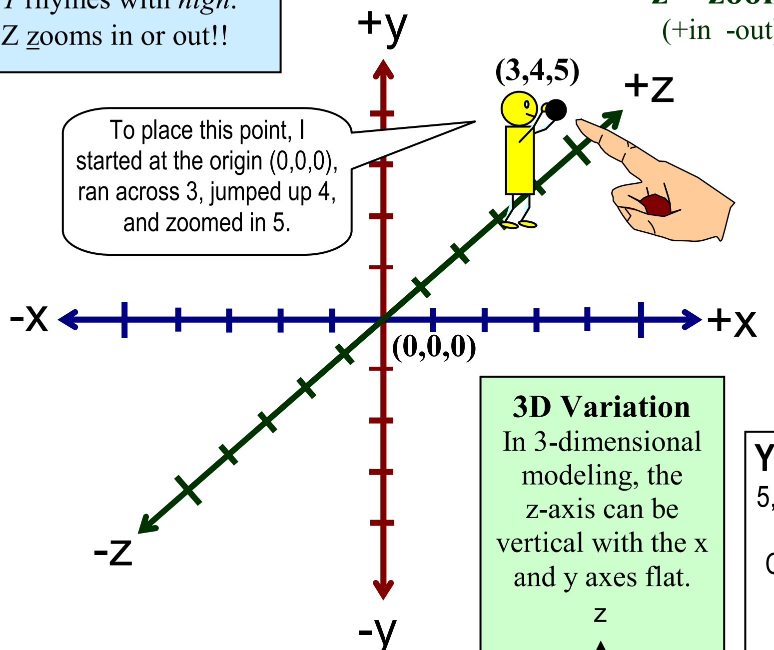
BrainAid

- X looks like "a cross."
- *Y* rhymes with *high*.
- Z zooms in or out!!

y = high
(+up -down)

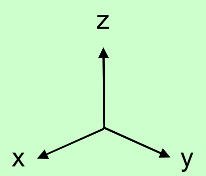
z = zoom
(+in -out)

x = across
(-left +right)

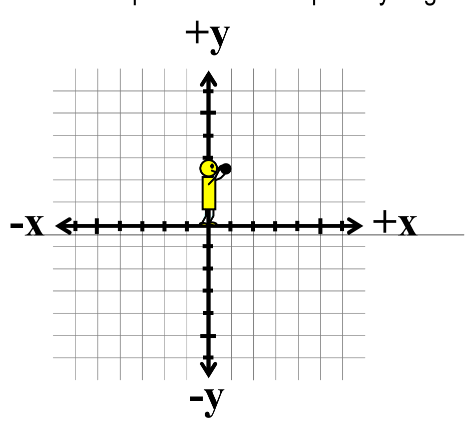


Paradox!
A point itself has no length, width, or height. It's dimensionless! It merely represents a *position* in two (x,y) or three (x,y,z) dimensions.

3D Variation
In 3-dimensional modeling, the z-axis can be vertical with the x and y axes flat.



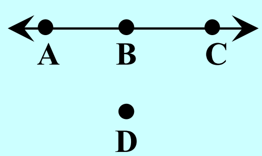
Your turn! Start at (0,0), run across 5, jump up 5, and place a point at (5,5). Starting at (0,0) each time, place points at (5,-5), (-5,5), and (-5,-5). Connect all four points. What shape do you get?



Collinear

Points that coexist on the same line.

A, B, and C are collinear. D is not.



4th Dimension?
The first 3 dimensions (x,y,z) tell us where something is. The 4th dimension (time) tells us *when* it's there!

Analytic Geometry

is the marriage of Algebra and Geometry, using coordinates for positions; e.g., the algebraic formula

y = mx + b

when graphed on a Cartesian plane, produces a straight line.

x, y = points on the line.
m = slope (steepness/direction) of line.
b = y-intercept (where line crosses y axis)

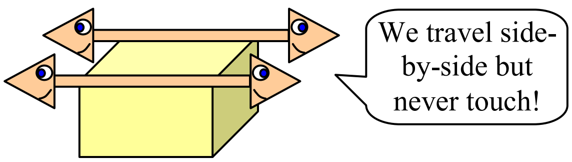
I invented Analytic Geometry so I could see how equations "looked." *Cartesian* [kar-TEE-zhun] planes (axes) and coordinates (points) are named after me!

René Descartes
[reh-naa daa-kart]
French * 1596-1650

Family Lines

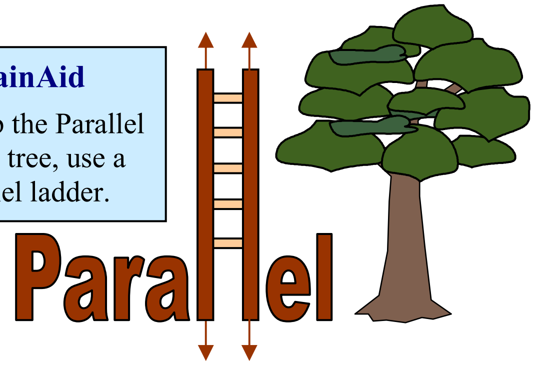
Parallel Line Family

Parallel [PAIR-uh-lel] lines live in the same plane but travel in the exact same direction, so they never touch.



BrainAid

To climb the Parallel Family tree, use a Parallel ladder.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

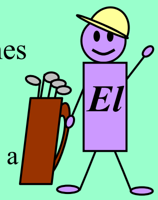
|| is the symbol for parallel lines.

TIP!

To spell *parallel*, imagine a golfer wishes his friend, El, good luck by saying:

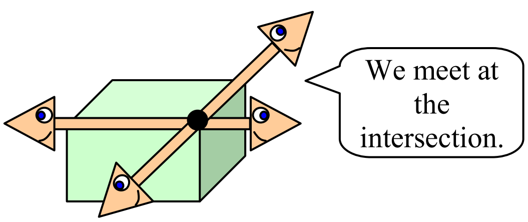
"Par all El."

"Par" is the number of strokes it should take a professional to hole the ball; e.g., Par 3.



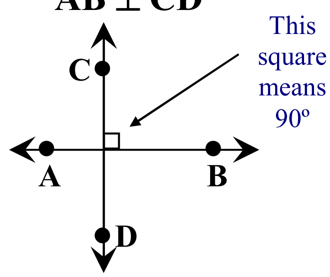
Intersecting Line Family

Intersecting lines live in the same plane, travel in different directions, and touch (intersect) at one point.



Perpendicular [pur-pen-DIK-yu-lur] lines intersect at 90º angles.

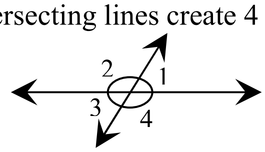
$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$



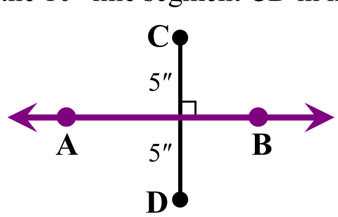
⊥ is the symbol for perpendicular lines.

Identical lines live in the same plane, travel in parallel directions, and intersect at *every* point.

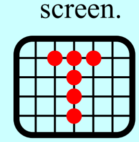
Intersecting lines create 4 angles.



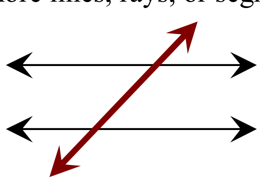
Perpendicular Bisector is a line (or segment, ray, plane) that intersects a line segment at a 90º angle, cutting it into two congruent parts. e.g., The Perpendicular Bisector $\overleftrightarrow{AB}$ cuts the 10″ line segment $\overline{CD}$ in half.



Geomancy is an ancient form of mystical geometry which claimed that forces concentrate at the intersections of lines. In modern flatscreen monitors, forces (pixels) concentrate at the intersections of lines (wires) embedded in the screen.

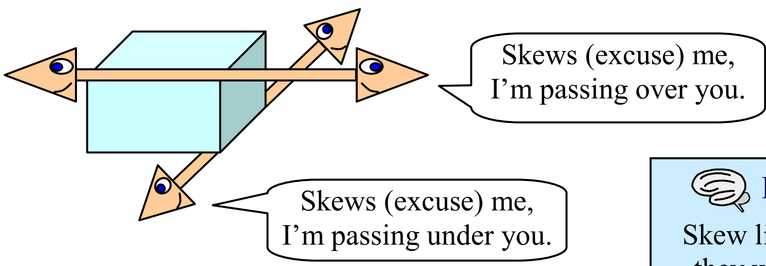


A **Transversal** [tranz-VERSS-ul] line "transfers all" of itself across 2 or more lines, rays, or segments.



Skew Line Family

Skew lines live in different planes, travel in different directions, and never touch.

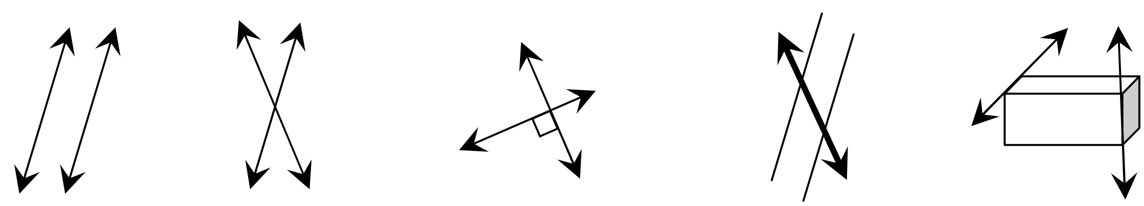


BrainAid

Skew lines are so polite, they wouldn't think of touching each other.

Your turn!

Label each line type.



Your turn!

Draw $\overleftrightarrow{EF}$ parallel to $\overleftrightarrow{GH}$.
Write an expression for this.

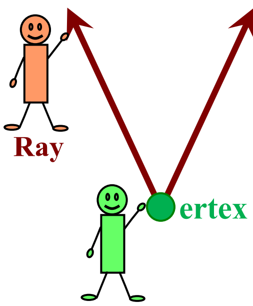
Your turn!

Draw $\overleftrightarrow{JK}$ perpendicular to $\overline{LM}$.
Write an expression for this.

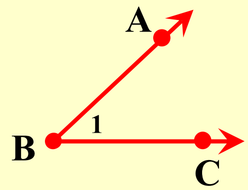
The Angle Boys

An angle [ANG-ul] is formed by two rays joined by a common endpoint called the *vertex*.
Angles are also formed when segments, lines, and other GeoParts intersect.

BrainAid
Imagine twins named Ray and their older brother, Vertex, creating an angle.

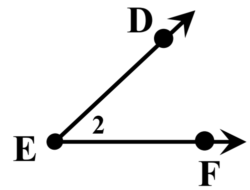


Ways to Name Angles ∠



- Using three points: ∠ABC or ∠CBA
- Using vertex letter: ∠B
- Using inside number: ∠1

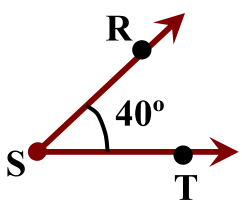
Your turn!
Use the angle symbol ∠ to list *four* names for the following angle.



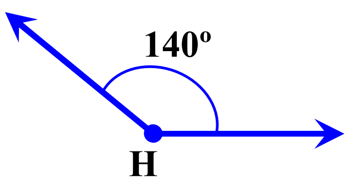
Angle Measure (m∠)

m∠
"measure of angle"

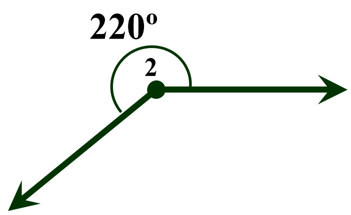
An angle is measured by the number of degrees (º) between its rays, from 0º to 360º.



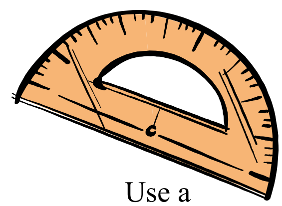
m∠RST = 40º



m∠H = 140º



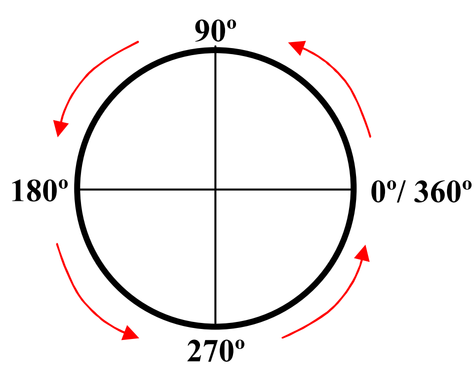
m∠2 = 220º



Use a protractor to measure angles.

Why 360º?

360-Day Year
The Babylonians chose 360 for the degrees in a circle, because it was the number of days they calculated the sun took to complete a circular year.



All-In-The-Box factoring shows that 360 has an amazing number of factor pairs: 12 in all!

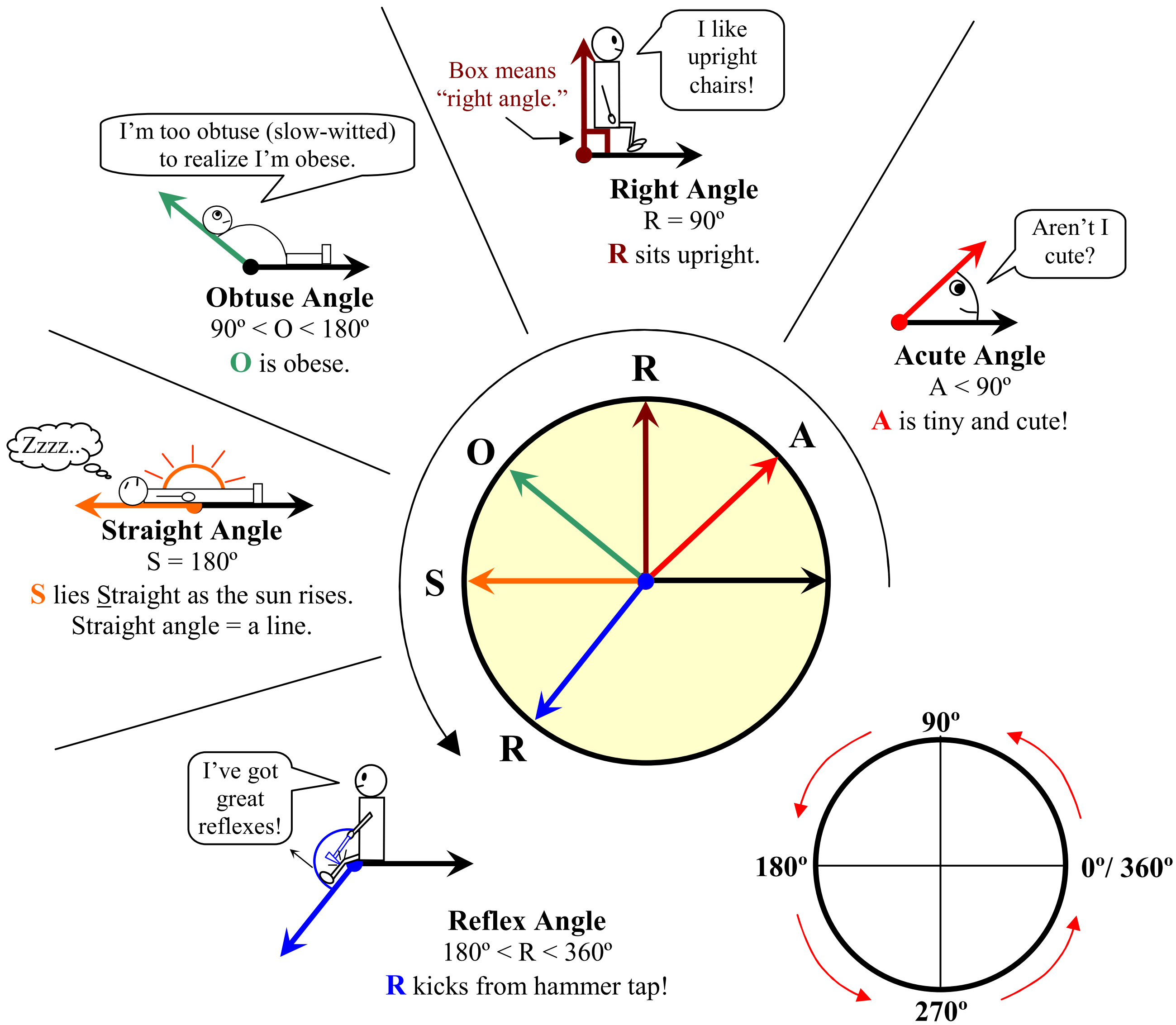
This means a circle can be easily divided into equal smaller pieces.

360

1	×	360
2	×	180
3	×	120
4	×	90
5	×	72
6	×	60
8	×	45
9	×	40
10	×	36
12	×	30
15	×	24
18	×	20

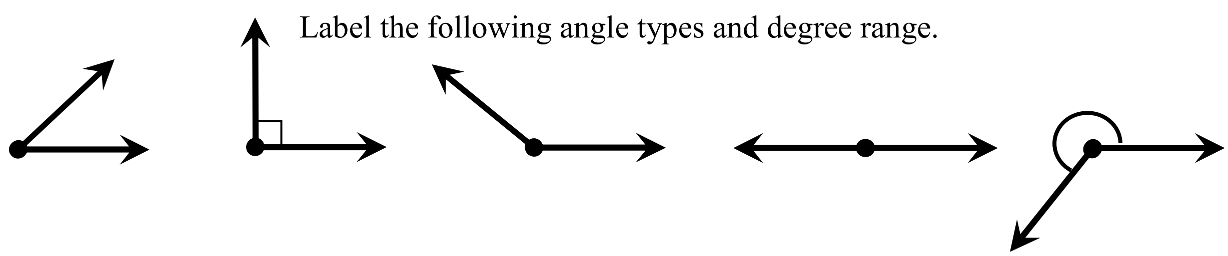
The AROSR Family

[uh-ROH-sir]



Your turn!

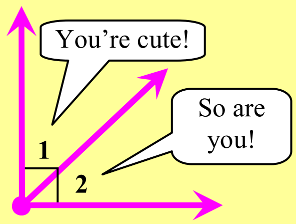
Label the following angle types and degree range.



Angular Relations

Complementary Angles (90º)

are "right" to "compliment" each other. [kawm-pluh-MEN-tur-ee]



m∠1 + m∠2 = 90º

BrainAids

Make "Co" into 90 by adding a line:

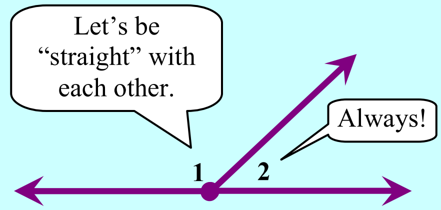
Complementary

Alphanumerically, C90 < S180

Comp
Supp
> lementary

Supplementary Angles (180º)

combine to make a Straight line. [suh-pluh-MEN-tur-ee]

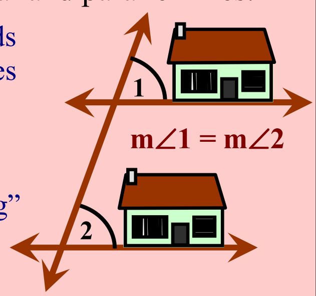


m∠1 + m∠2 = 180º

Corresponding Angles

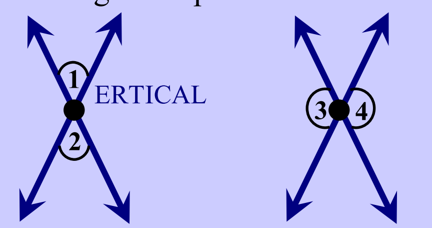
reside on the same "corner" of transversal and parallel lines.

Imagine friends living in houses on the same corner of the next block "corresponding" via email.



Vertical Angles

reside on opposite sides of a vertex, making V-shapes on all four sides.



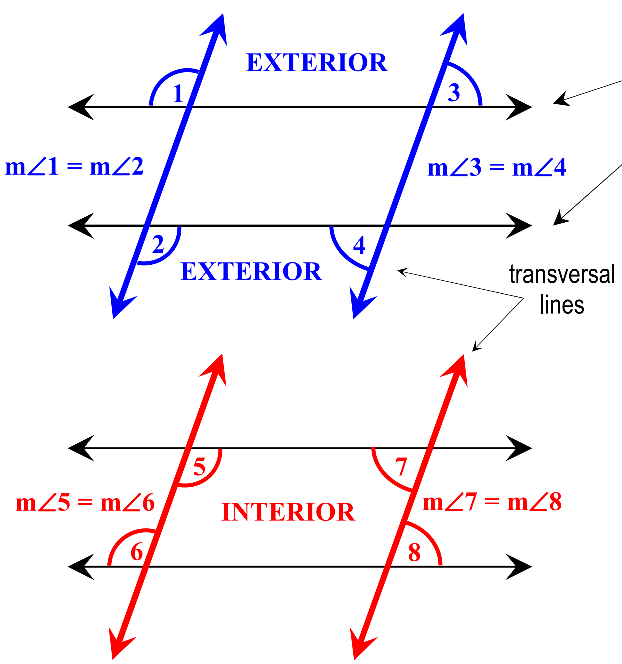
m∠1 = m∠2 **m∠3 = m∠4**

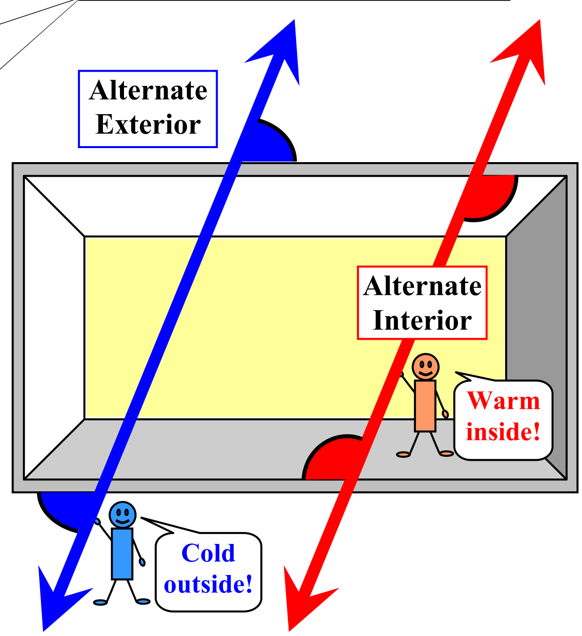
Alternate Angles

reside on opposite sides of transversals and on the exterior (outside) or interior (inside) of parallel lines.

BrainAid

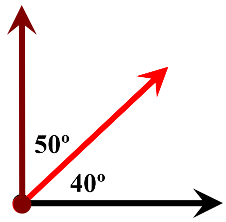
If you pick up one parallel line and place it on top of the other, you can see that ∠1 & ∠2 are equal vertical angles, as are ∠3 and ∠4.

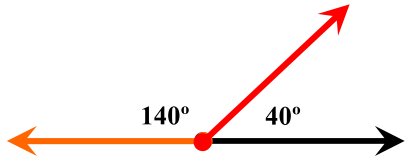


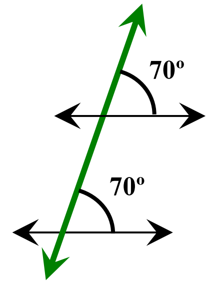


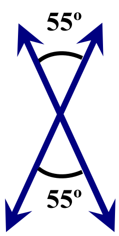
Your turn!

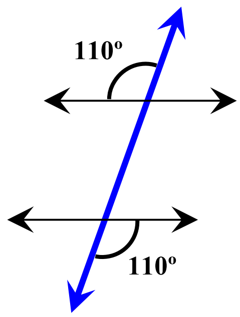
Label the following angular relations.

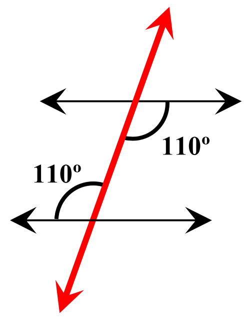






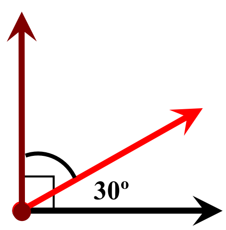


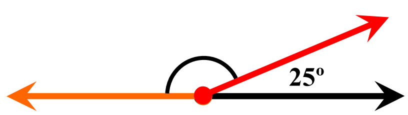


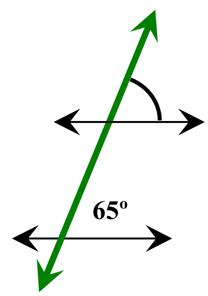


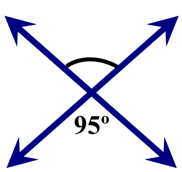
Your turn!

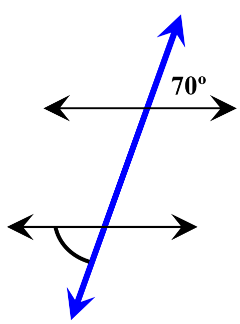
Fill in the missing angle degrees.

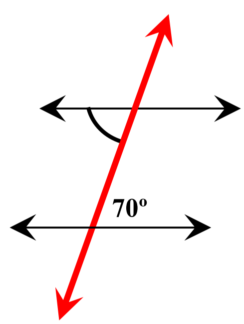






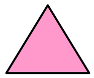
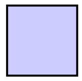
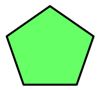
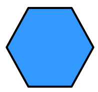






Polygons

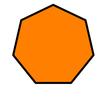
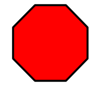
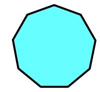
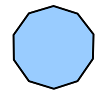
Polygons are closed figures formed by line segments that create angles.
Each intersection of line segments is a **vertex.** The plural of vertex is **vertices.**

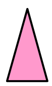
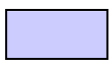
Name	Figure
Triangle (3 angles)	
Quadrilateral (4 sides)	
Pentagon (5 angles)	
Hexagon (6 angles)	

Poly **= many**
gon **= angle**
lateral **= side**

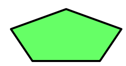
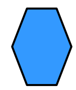
The number of sides equals the number of angles.

Regular Polygons
All sides/angles congruent.

Name	Figure
Heptagon (7 angles)	
Octagon (8 angles)	
Nonagon (9 angles)	
Decagon (10 angles)	

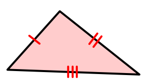
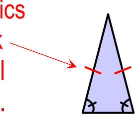
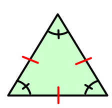
 

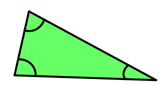
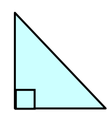
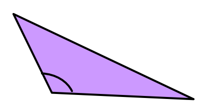
Irregular Polygons
Not all sides/angles congruent.

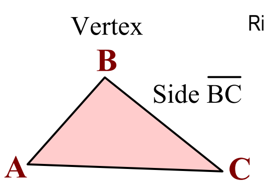
Triangles

Triangles are polygons with three sides, three angles, and three vertices.

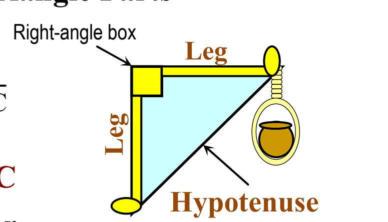
Side Classifications (S I d E)		
Name	**Congruent Sides**	**Example**
Scalene skalenos = uneven	0	
Isosceles Iso = equal skeles = legs	2	Equal tics mark equal parts. 
Equilateral Equal sides (aka Equiangular)	3	

Angle Classifications (A R O s r)		
Name	**Angle/s**	**Example**
Acute	All < 90º	
Right	1 = 90º	
Obtuse	1 > 90º	

Triangle Parts

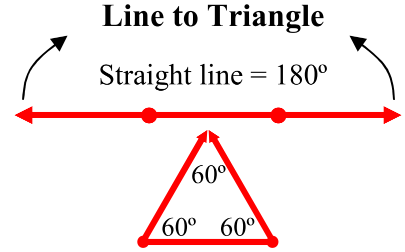


Triangle ΔABC or ΔBCA or ΔCAB



Imagine a "high pot in noose" hanging opposite a right triangle's legs & "box"er shorts.

Line to Triangle

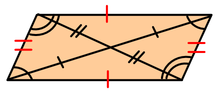
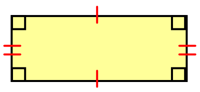
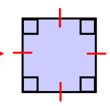
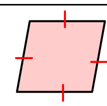
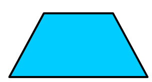


When a 180º line is folded into a triangle, the inside angles *always* add up to 180º.

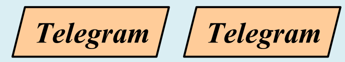
Quadrilaterals

Quadrilaterals are polygons with four sides, four angles, and four vertices.

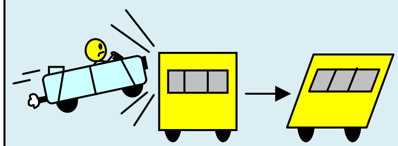
Types of Quadrilaterals

Name	Features	Figure
Parallelogram	• Opposite sides parallel. • Opposite sides congruent. • Opposite angles congruent. • Diagonals bisect.	
Rectangle	Special Parallelogram • All right angles. (A square is also a rectangle)	
Square	Special Parallelogram • All right angles. • All sides congruent.	Equal tics mark equal parts. 
Rhombus	Special Parallelogram • All sides congruent. (A square is also a rhombus)	
Trapezoid	Quadrilateral • One set of parallel sides.	

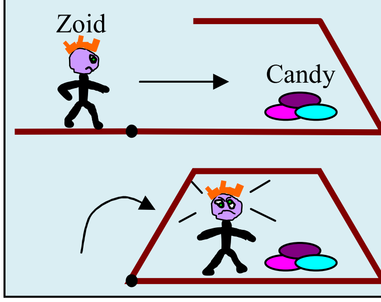
Pair-a-telegrams!



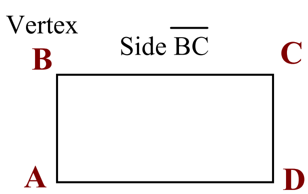
Rhom [into a square] **bus!**



How to Trap e Zoid!



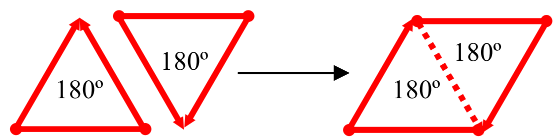
Quadrilateral Parts



Rectangle ABCD, BCDA, etc.

360º

A quadrilateral can be made from two triangles, each with 180º. Twice 180º is 360º



The inside angles of a quadrilateral add to 360º.

Interior Angles

Sum of Interior Angles of a Polygon

$(n-2) \times 180^\circ$

n = number of sides (or angles)

Triangle: $(3 - 2)\times180^\circ = 1\times180^\circ = 180^\circ$

Quadrilateral: $(4 - 2)\times180^\circ = 2\times180^\circ = 360^\circ$

Pentagon: $(5 - 2)\times180^\circ = 3\times180^\circ = 540^\circ$

Measure of one Interior Angle of a *Regular* Polygon

$$\frac{\text{Sum of Interior Angles}}{\text{Number of Angles}} = \frac{(n-2)\times 180^\circ}{n}$$

Pentagon: 540º / 5 = 108º per angle

BrainAid

Knock **2** sides off a triangle to get a **180º** line.

Your turn!

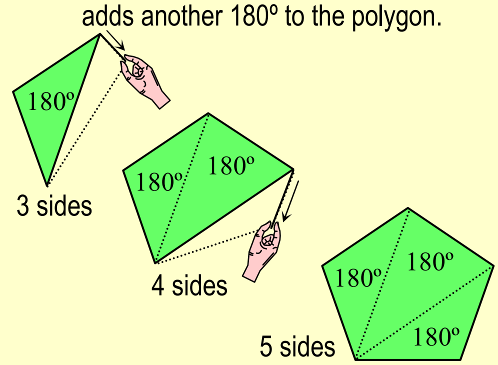
Regular Hexagon

Sum interior angles

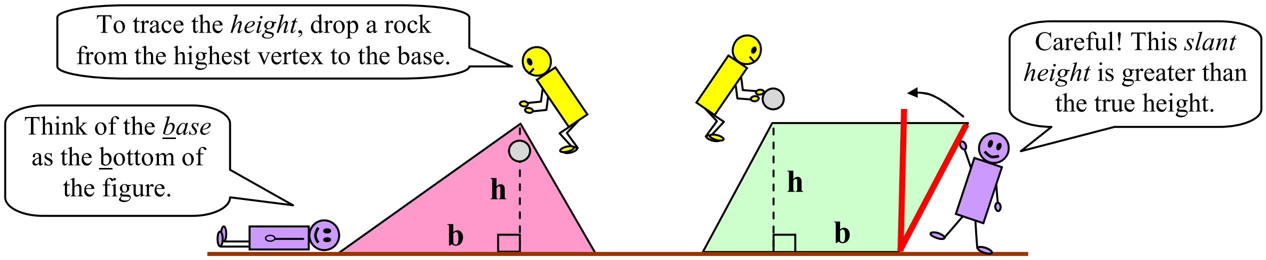
Size of one angle

Why it works

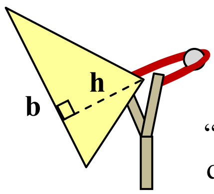
(n - 2) removes two of the triangle's sides, so it's again equivalent to a line of 180º. Each "pull" on a vertex adds 1 new side (line) which adds another 180º to the polygon.



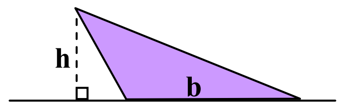
Measuring Polygons



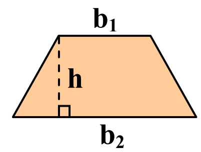
It's easier to work with polygons if the base **b** is on the bottom.
But in all cases the height **h** must be at a 90º angle to the base.



For tilted objects, "slingshot" the rock directly to the base.



The height can be *outside* the figure.



A trapezoid has *two* bases.

Perimeter of Polygon

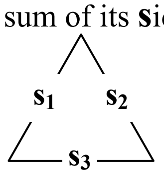
Perimeter [pur-RIM-eh-tur] is a measure of the distance around an object.
Peri = around *meter* = measure

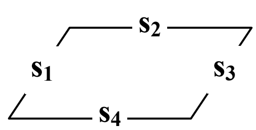
Perimeter of Polygon

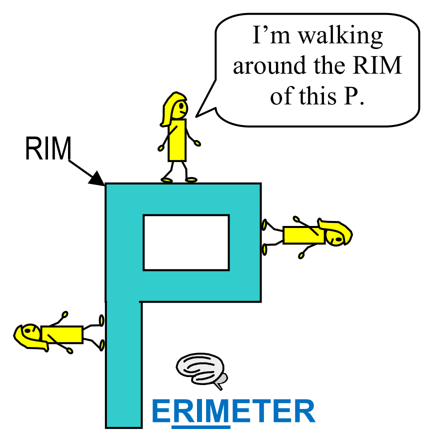
The perimeter of a polygon is the sum of its **s**ides.

$\mathbf{P}_{\text{triangle}} = s_1 + s_2 + s_3$

$\mathbf{P}_{\text{quadrilateral}} = s_1 + s_2 + s_3 + s_4$







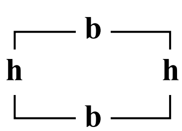
Perimeter of Rectangle

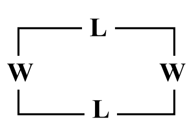
The perimeter of a rectangle is twice its **b**ase plus twice its **h**eight.

$\mathbf{P}_{\text{rectangle}} = 2b + 2h$

or

$2(b + h)$



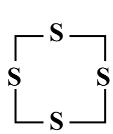


Alternate Variables:
L=Length (long side)
W=Width (short side)

Perimeter of Square

The perimeter of a square is four times the length of one **s**ide.

$\mathbf{P}_{\text{square}} = 4s$



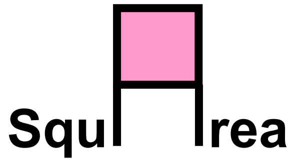
Your turn!

How much fencing is needed to enclose a 7 ft by 5 ft yard?

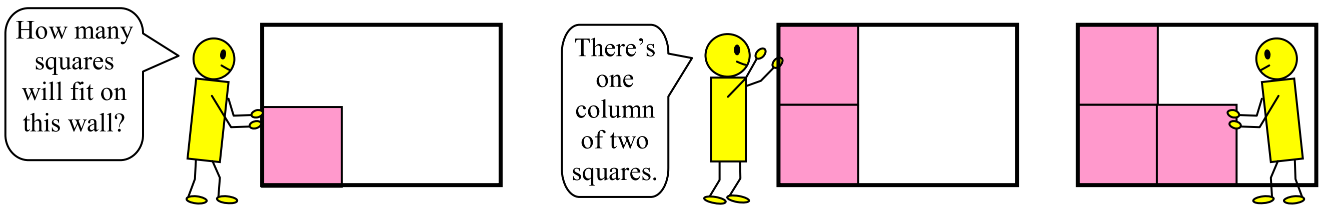
Area of Polygon

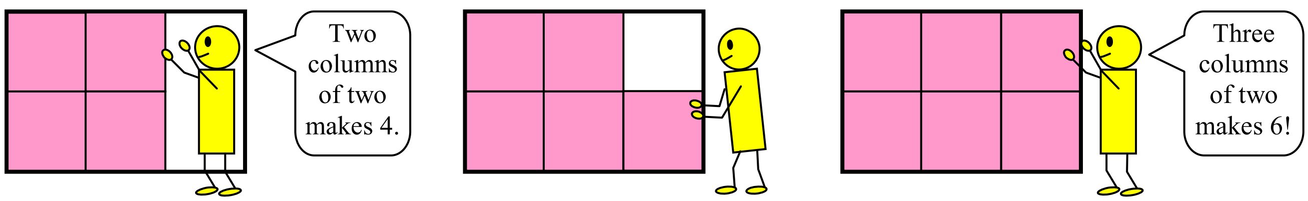
Area [AIR-ee-uh] is the number of *squares* that will fit on the surface of the polygon.
Area is Latin for “level ground” or “open space.”

BrainAid
Square → SquArea ← Area



BrainAid
Imagine that the top of the “A” in SquArea is a square.

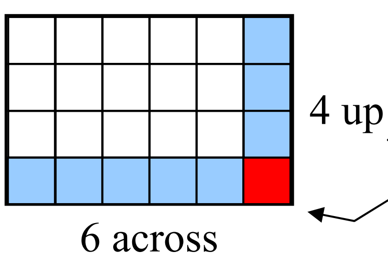




Shortcut

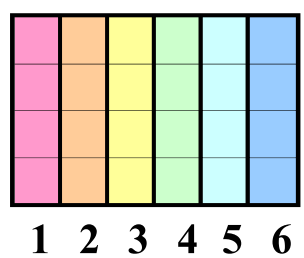
To calculate the number of squares that fit in a rectangle, multiply the number of squares across the bottom times the number of squares up one side!

6 × 4 = 24 squares



Paradox!
Multiplying this way makes it seem like you’re counting the corner square twice!

TIP!
Think in terms of columns; in this case, 6 four-high columns!



Units of Measure

Lengths are measure in linear units: e.g., inches.

Since Areas multiply length × length, they are measured in *square* units: e.g., square inches (in^2)

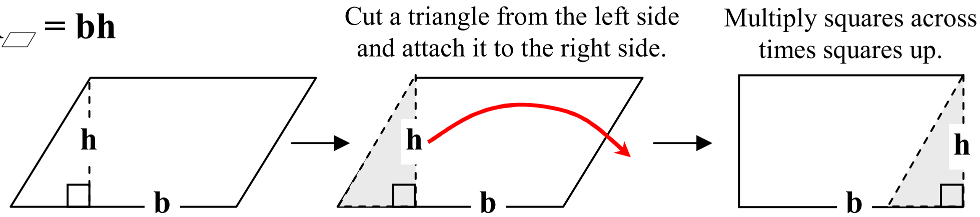
Your turn!

How many square feet of sod will cover a 7 ft by 5 ft field?

Area of Parallelogram

Since a parallelogram can be made into a rectangle, its area is base times height.

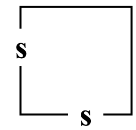
$A_{\text{▱}} = \mathbf{bh}$



Parallelogram Features: Opposite angles are congruent; Diagonals bisect each other.

$A_{square} = s^2$

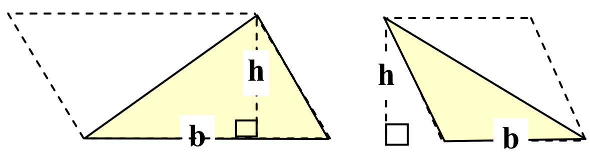
The area of a square is the length of one side squared.



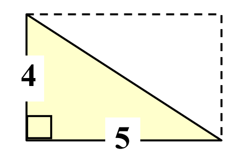
Area of Triangle

Since a triangle is half a parallelogram, its area is ½ base times height.

$A_{\triangle} = \frac{1}{2}\mathbf{bh}$



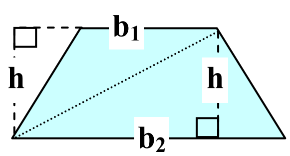
Your turn!
How many squares will fit in this triangle?



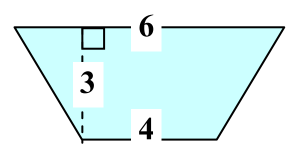
Area of Trapezoid

Since a trapezoid can be split into two triangles, its area is a combination of both.

$A_{\text{⏢}} = \frac{1}{2}b_1h + \frac{1}{2}b_2h = \mathbf{\frac{1}{2}(b_1 + b_2)h}$



Your turn!
How many squares will fit in this trapezoid?

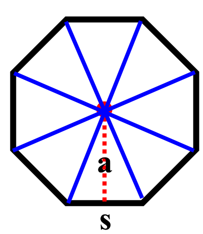


Area of Regular Polygon

Since a regular polygon can be split into triangles, its area is equivalent to the sum of the areas of all triangles inside it. The area of one internal triangle is **½ sa** where **s** = side of polygon (base) and **a** = apothem (height). The sum of all sides of the polygon is its Perimeter $P = s_1 + s_2 + s_3$.... Therefore the area of all triangles in a polygon would be ½ **P**erimeter times **a**pothem.

$A_{\text{⬠}} = \mathbf{\frac{1}{2}\ Pa}$

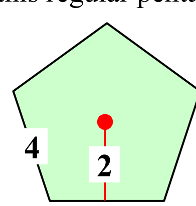
Apothem [A-puh-thum]
The line segment from the center of a regular polygon to the midpoint of a side.



The apothem is the height of every internal triangle.

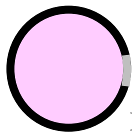
The perimeter is the sum of the sides which make up the bases of all the triangles.

Your turn!
How many squares will fit in this regular pentagon?

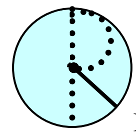


Circles

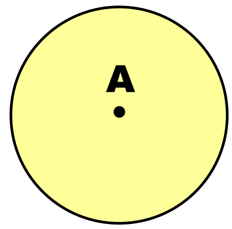
A circle is a set of points equidistant from a center point.


ircumference

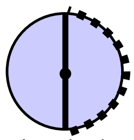
Imagine the letter
C circling the
perimeter.


adius

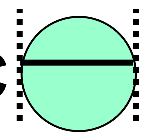
Imagine the leg of the letter
'R' radiating from the center.
2 × radius = Diameter



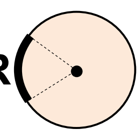
Circle A


iameter

Imagine the letter D dividing the
circle through its center. In Greek:
dia=across; meter=measure.
Diameter = 2 × radius

C**ord**

Imagine the crossbar of H in cHord as a
line segment crossing the circle.
Any chord that passes through the
center is also a Diameter.

AR

Imagine the small c in arc is a part circle.

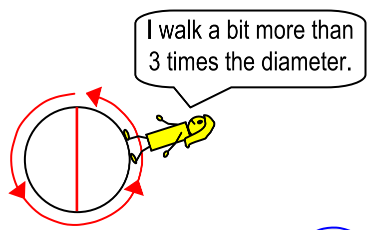
Circumference of Circle

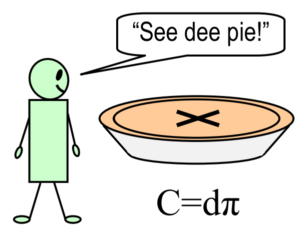
Circumference equals diameter times π (pi).

$C = d\pi$ [$\pi = \sim 3.14$ or $\sim 22/7$]

Alternate formula: $C = 2\pi r$

Early mathematicians discovered that the distance
around *any* circle was just over 3 times its diameter.
They named this ratio "pi" (Greek for periphery).





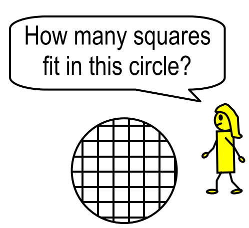
$C=d\pi$

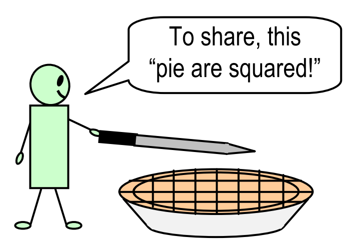
BrainAids

Area of Circle

Area equals π times radius squared.

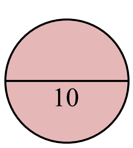
$A = \pi r^2$





$A=\pi r^2$

Your turn!
What is the distance around a
circle whose diameter is 10?



Your turn!
How many squares will fit in
a circle whose radius is 10?