Max Learning’s Algebra Antics

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Welcome!

Hi, my name is Max Learning, and I’ll be your teacher and guide.
My goal is to make math seem “real” to you, so you’ll gain confidence and look forward to your next math challenge.
The fact that you’re reading this book means you’re eager to succeed and are willing to explore new ways to do so. So let’s get started!

Why Is Math A Struggle?  How This Book Can Help

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Mental Manipulatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math uses symbols, lots of them. It’s as difficult to learn as a foreign language.</td>
<td>You’ll learn to “see” three-dimensional objects behind each symbol.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math is based on rules, lots of them. It's hard not to confuse one for the other.</td>
</tr>
<tr>
<td>BrainAids</td>
</tr>
<tr>
<td>You’ll learn clever memory hints that make the rules easy and fun.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Trauma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting an answer wrong in front of the class, losing at a flash-card competition, failing a test, being criticized by a teacher—all can lead to math trauma.</td>
</tr>
<tr>
<td>RUFF</td>
</tr>
<tr>
<td>You’ll learn to be in a Relaxed, Uncluttered, Focused, and Flowing state of mind, which increases confidence and eases past traumas.</td>
</tr>
</tbody>
</table>

What’s Good About Math?

Certainty
Math problems have right answers. In most subjects, like English or Art, the grade you get on an essay or project depends on your teacher’s opinion of your work. However, in a Math class, when you get the right answer, no one can argue with it. It’s certain!

Quest
Math problems are puzzles. The quest to solve them can be exciting! If you approach it with this attitude, math can be as fun and engaging as any game you’ll ever play. Solving problems that others find difficult is very satisfying and makes you feel smart!

Magic
Math is the language of nature. Nearly everything we see, hear, touch and do can be described with math. Every image, color, and sound on a computer is based on math. In today’s movies, you can’t always tell what’s real and what’s been generated by some mathematical formula. In short, math is amazing—there’s magic in it!
Note to Parents

I’ve kept the problems in this book simple, so you and your kids could grasp the concepts without getting bogged down in the arithmetic. And I’ve tried to make it as interesting and memorable as possible with illustrations, Mental Manipulatives, and BrainAids.

But don’t be surprised if your kids don’t rush to do math on their own. Except for the rare few who find it fun and challenging, most avoid math like the plague. After all, it’s not always easy, and most of us avoid uncomfortable mental effort whenever possible.

But math is a school requirement, students have to learn it, so I try to make it as painless as possible. And many children, once they “see the light” and have tasted success, come to enjoy the subject.

If your child is not motivated to read this book, or has trouble understanding some of the concepts or techniques, I recommend you first learn them on your own, then teach them to your child. It’s what I would do in a classroom or tutoring session. I only wrote the book because I can’t be everywhere to teach every student. Besides, most of us would rather be shown how to do something rather than having to read about it.

This is a techniques book rather than a drill & practice book. Check your answers to the Your turn activities in the Answer Key in the back of the book. Then apply these techniques to the numerous problems in traditional math textbooks, or try to make up some problems of your own.

You’re learning a new, I hope, more interesting way of doing math. As with learning anything new, it’s best not to rush; so relax, take your time, and enjoy the process!

Pronunciation Guide

Sometimes it may not be obvious how to pronounce terms you have not heard spoken. When you see a term followed by a pronunciation, refer to this guide as needed.

<table>
<thead>
<tr>
<th>Vowels</th>
<th>Consonants</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long</strong></td>
<td><strong>Hard</strong></td>
</tr>
<tr>
<td>aa = ate</td>
<td>k = cat</td>
</tr>
<tr>
<td>a = act</td>
<td>g = go</td>
</tr>
<tr>
<td>ee = eel</td>
<td>s/ss = hiss</td>
</tr>
<tr>
<td>e/ch = end</td>
<td>ch = chin</td>
</tr>
<tr>
<td>ii = hi</td>
<td>th = thin</td>
</tr>
<tr>
<td>i/ih = hid</td>
<td></td>
</tr>
<tr>
<td>oh = no</td>
<td>oo = book, or = for</td>
</tr>
<tr>
<td>aw = on</td>
<td>ow = how, oy = boy</td>
</tr>
<tr>
<td>uu = too, ur = fur</td>
<td></td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td><strong>Accent on:</strong> UP-ur-KAASS</td>
</tr>
</tbody>
</table>

Common Abbreviations

aka = also known as

e.g. = for example (think ezample)

i.e. = that is

p. = page

FYI = For Your Information
It was a mouthful to say *mnemonic* (nee-MAWN-ik) *device*, so I coined the word *BrainAid* for memory hints that help you learn. Feel free to make up your own BrainAids. Most fall into one of the following "A" categories:

**Analogy = Comparison**

*How to say it:* uh-NOWL-uh-jee

*What it is:* A *comparison* of what you are trying to learn to what you already know.

*Why it works:* To learn, new brain fibers must grow and laboriously push their way through dense brain tissue, which can be very tiring. An analogy lets you piggyback new knowledge on top of *existing* brain fibers, which is quicker and takes much less effort.

*Analogy Example:* Just as *physical* exercise builds new *muscle* fibers, *mental* exercise builds new *brain* fibers. Both take time, effort, and repetition.

**Acronym = Name**

*How to say it:* AK-roh-nim

*What it is:* A *name* made from the first letters of several words. Hint: Think *nym* = *name*.

*Why it works:* The letters of an acronym act like small hooks on which you can hang large words. You memorize a little to remember a lot.

*Acronym Example:* To maximize your learning, be in a learning frame of mind: Relaxed, Uncluttered, Focused, and Flowing. In other words, be RUFF.

**Acrostic = Story**

*How to say it:* uh-KRAW-stik

*What it is:* A *story* made from the first letters of several words. Hint: Think *stic* = *story*.

*Why it works:* Sometimes a "story" is easier to remember than an acronym, especially if the acronym is hard to pronounce.

*Acrostic Example:* You create the acronym MTF to help you remember the names of three new acquaintances, Mary, Tom, and Fred. But MTF is hard to pronounce, so you convert it into the acrostic "*My Three Friends*."
Concepts

Math Basics

In Max Learning’s Mental Math and Fraction Fun books, we learned several concepts that will help us in Algebra Antics. Please refer to these books for more details on the following Math Basics concepts.

Mental Manipulatives

Traditional manipulatives are physical objects, like tiles or blocks, which you “manipulate” to mimic math operations. Mental manipulatives are items you visualize when you see a number or operation. They can turn lifeless symbols into reality—at least in your imagination.

And unlike physical manipulatives, mental manipulatives are always with you to make math more engaging. Mental Manipulatives include piles, holes, MathBots, and many other items.

Numbers

A number is a symbol for a quantity or value.

Natural Numbers: Counting numbers: 1, 2, 3...

Whole Numbers: Zero + Natural numbers: 0, 1, 2, 3...

Integers: Negatives of Natural numbers + Whole numbers: ...-3, -2, -1, 0, 1, 2, 3...

Rational Numbers: Can be written as ratios. Consist of integers, fractions, terminating or repeating decimals: 2, ½, .33

Irrational Numbers: Can not be written as ratios. Consist of non-repeating or non-terminating decimals: π, √2

Real Numbers: All rational and irrational numbers.

Imaginary Number: √-1 or i

Complex Numbers: Real number with imaginary number: 3 + i
Operators & Operands

An operator is a symbol for a procedure or relationship between operands. Operands include: addends, minuends & subtrahends, multipliers & multiplicands, dividends & divisors.

<table>
<thead>
<tr>
<th>Arithmetic Operators</th>
<th>Relational Operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic operators specify procedures.</td>
<td>Relational operators specify relationships.</td>
</tr>
<tr>
<td>+ Add</td>
<td>= Equal</td>
</tr>
<tr>
<td>− Subtract</td>
<td>≠ Not equal to</td>
</tr>
<tr>
<td>× • Multiply</td>
<td>&gt; Greater than</td>
</tr>
<tr>
<td>÷ / Divide</td>
<td>&lt; Less than</td>
</tr>
<tr>
<td>± Plus or Minus</td>
<td>≥ Greater than or equal to</td>
</tr>
<tr>
<td>≤ Less than or equal to</td>
<td></td>
</tr>
</tbody>
</table>

Computer Operators

Many of the common operators do not appear on computer keyboards. Below are alternates, typically used in computer spreadsheet formulas.

<table>
<thead>
<tr>
<th>Computer Operators</th>
<th>BrainAid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic operators specify procedures.</td>
<td>Be careful not to confuse the &gt; and &lt; symbols. The larger number goes on the larger side.</td>
</tr>
<tr>
<td>Many of the common operators do not appear on computer keyboards. Below are alternates, typically used in computer spreadsheet formulas.</td>
<td>Example: 7 &gt; 6; 6 &lt; 7</td>
</tr>
<tr>
<td>* Asterisk (aka star) for multiply</td>
<td>LARGE MOUTH &gt; Small throat</td>
</tr>
<tr>
<td>^ Caret [KAIR-et] for exponentiation.</td>
<td>Small throat &lt; LARGE MOUTH</td>
</tr>
<tr>
<td>&lt;&gt; Not equal to</td>
<td>≥ Greater than or equal to</td>
</tr>
<tr>
<td>&gt; Greater than or equal to</td>
<td>≤ Less than or equal to</td>
</tr>
<tr>
<td>&lt;= Less than or equal to</td>
<td></td>
</tr>
</tbody>
</table>

Algorithms


Algorithms make math operations easier.

Instead of having to figure out what to do each time, you follow the algorithm.

For example, the procedure you follow when doing long division is an algorithm.

<table>
<thead>
<tr>
<th>BrainAid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al go(t) rithm.</td>
</tr>
<tr>
<td>He smartly follows the dance’s step-by-step procedure.</td>
</tr>
</tbody>
</table>

I’ve got rhythm!

Step, Step, Step
**PEMDAS**  
Priority of Operations

When a math problem has more than one operator, work in this order:

- **Parentheses**: Perform operations inside of parentheses first. If nested, start with the innermost set of parentheses: ( Do 2nd (do 1st) ).

- **Exponentiation**: Raise numbers to powers.

- **Multiplication/Division**: If encounter both, perform in left-to-right order.

- **Addition/Subtraction**: If encounter both, perform in left-to-right order.

<table>
<thead>
<tr>
<th>Parentheses Package</th>
<th>Exponentiation Expands</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 - (2 + 1)$</td>
<td>$2^2$</td>
</tr>
<tr>
<td>$5 - 3$</td>
<td>$4$</td>
</tr>
<tr>
<td>$2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiplication Magnifies</th>
<th>Division Dissolves</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 3$</td>
<td>$4 \div 2$</td>
</tr>
<tr>
<td>$6$</td>
<td>$2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Addition Attaches</th>
<th>Subtraction Steals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 + 1 = 4$</td>
<td>$3 - 1 = 2$</td>
</tr>
</tbody>
</table>
Factors

Factors are multipliers that combine to make products.

\[ \text{Factor} \times \text{Factor} = \text{Product} \]

Example: \(2 \times 3 = 6\), so 2 and 3 are factors of the product 6.

Factoring is the process of finding a product’s factors.

To factor means to extract the multipliers that form a product.

Example: 6 can be factored into \(1 \times 6\) or \(2 \times 3\), so the factors of 6 are 1, 2, 3, 6.

Common Factors are factors that are the same for different products.

<table>
<thead>
<tr>
<th>Product</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1, 2, 3, 4, 6, 12</td>
</tr>
<tr>
<td>20</td>
<td>1, 2, 4, 5, 10, 20</td>
</tr>
</tbody>
</table>

1, 2, and 4 are common factors of the products 12 and 20.
4 is the Greatest Common Factor (GCF) of 12 and 20.

Factoring Tricks

Use these tricks to see if a number contains a factor before you waste time trying to extract it.

A product is evenly\(^*\) divisible by a factor of:

- 2—If the product is even (i.e., ends in 0, 2, 4, 6, or 8).
- 3—If the sum of the product’s digits is a multiple of 3 (321: 3+2+1 = 6).
- 4—If the product’s last 2 digits are a multiple of 4 (316).
- 5—If the product ends in 0 or 5 (765).
- 6—If the product fits the tricks for both 2 and 3 above (462: 4+6+2 = 12).
- 7—If the product’s 1\(^{st}\) digits minus (2 \times the last digit) is 0 or multiple of 7 [112: 11–(2\times2) = 11 – 4 = 7].
- 8—If the product’s last 3 digits are 000 or a multiple of 8 (2104).
- 9—If sum of the product’s digits is a multiple of 9 (864: 8+6+4 = 18).

\(^*\) Technically, every number is divisible by every number (except 0), but may not be evenly so; e.g., \(10\div4 = 2\frac{1}{2}\)

Composite factors are divisible by 1, themselves, and at least one other number.

Example: 4 is divisible by 1 and 4, but also by 2.

Prime factors are divisible by 1 and themselves only.

Example: 2 is divisible by 1 and 2 only. The same is true for 3, 5, 7, 11, etc.

0 and 1 by definition are neither composite nor prime.

Tip: To ensure complete factoring, factor until all factors are prime numbers.
**Factor Trees**

Factor Trees are useful for extracting prime factors.

**Tropical Factor Tree**

Imagine being on an island with a palm tree containing a coconut. Being hungry, you grip and shake the tree. As the coconut falls, it conveniently splits in smaller pieces full of prime nutrients for you to eat.

<table>
<thead>
<tr>
<th>Grip and shake the tree to dislodge the coconut (the product of your labor).</th>
<th>As it falls, the coconut splits into smaller pieces (factors).</th>
<th>On the way down, all pieces split into their prime nutrients.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Grip and shake" /></td>
<td><img src="image2.png" alt="As it falls" /></td>
<td><img src="image3.png" alt="On the way down" /></td>
</tr>
</tbody>
</table>

**Traditional Factor Tree**

To create a Factor Tree without having to call on your artistic ability:

- Draw two branches beneath the product to be factored.
- Extract the smallest prime factor (2, 3, 5, etc.) and place it under the left branch with the composite factor under the right branch.
- Repeat the process with the composite factor until all factors are prime.
- Box the prime numbers at the bottom of the branches.

**GCF: Grip, Catch, Focus**

To find the GCF of several products:

- Grip each products’ Factor Tree, and shake out its prime factors.
- Catch (circle) factors that are common to all products each time they occur.
- Focus on and magnify (multiply) one set of circled factors to get the GCF.

**Example:** Find the GCF of 8, 12, and 16.

- Grip each products’ Factor Tree, and shake out its prime factors.
- Catch (circle) factors that are common to all products each time they occur.
- Focus on and magnify (multiply) one set of circled factors to get the GCF.

**Example:** Find the GCF of 8, 12, and 16.

- Grip each products’ Factor Tree, and shake out its prime factors.
- Catch (circle) factors that are common to all products each time they occur.
- Focus on and magnify (multiply) one set of circled factors to get the GCF.

- GCF = 2 × 2 = 4

Observe that there are two 2s that are common to all products, so they are both circled.
Observe that only one set of common factors is multiplied to find the GCF.
Example of use: Extracting the GCF of 4 from 8, 12, and 16 reduces them to 2, 3, and 4 respectively.
Multiples

Multiples are *products* created by multiplying a base number times a series of numbers.

**Base × Number = Multiple**
Example: $2 \times 3 = 6$, so 6 is a multiple of base 2.

### Common Multiples

Are multiples that are the same for different bases.

<table>
<thead>
<tr>
<th>×</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base 2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Base 3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

6 and 12 are common multiples of the bases 2 and 3. 6 is the Least Common Multiple (LCM) of 2 and 3. LCM is also known as the *Lowest* Common Multiple.

In fractions, the LCM is the LCD: Least Common Denominator.

### Why Make Multiples?

One reason is to find a common number that several bases will dissolve into; e.g., a common denominator.

### LCM: Load, Crush, Mix

To find the LCM of several products, factor each product into prime factors (p.10).
- **Load** all of the first product’s prime factors into a large cooking pot.
- **Crush** (cross out) factors from the next product/s that are already in the pot. Load what’s left.
- **Mix** (magnify/multiply) the factors in the cooking pot to get the LCM.

**Example:** Find the LCM of 8, 12, and 16.

The goal is to not include any more factors than are needed to dissolve each product into the cooking pot. Crushing eliminates redundant factors.

To find the LCM:
- Load: $2 \times 2 \times 2$ for 8.
- Crush: $2 \times 2$ for 12.
- Mix: $2 \times 2 \times 3$ for 16.

**LCM = 2 × 2 × 2 × 3 × 2 = 48**

Dissolving 8, 12, and 16 into the LCM of 48 results in 6, 4, and 3. With denominators, the LCM is the LCD: Least Common Denominator.

Example of use: $\frac{1}{8} + \frac{1}{12} + \frac{1}{16} = \frac{6}{48} + \frac{4}{48} + \frac{3}{48} = \frac{13}{48}$.

Imagine multiples as mounds built from a base.
Algebra Basics

Term → Expression → Equation

Let’s compare what you already know about English parts of speech to Algebra terminology.

<table>
<thead>
<tr>
<th>ENGLISH</th>
<th>ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Word</strong></td>
<td><strong>Term</strong></td>
</tr>
<tr>
<td>John</td>
<td>1</td>
</tr>
<tr>
<td>conjunction</td>
<td>+ or – operator</td>
</tr>
<tr>
<td>and</td>
<td>+</td>
</tr>
<tr>
<td><strong>Phrase</strong></td>
<td><strong>Expression</strong></td>
</tr>
<tr>
<td>John and Mary</td>
<td>1 + 1</td>
</tr>
<tr>
<td><strong>verb</strong></td>
<td>relational operator</td>
</tr>
<tr>
<td>are</td>
<td>=</td>
</tr>
<tr>
<td><strong>Sentence</strong></td>
<td><strong>Equation</strong></td>
</tr>
<tr>
<td>John and Mary are together.</td>
<td>1 + 1 = 2</td>
</tr>
</tbody>
</table>

**TERM**

A term is a mathematical word.

The + or – operators are mathematical *conjunctions* that join terms.

**EXPRESSION**

An expression is a mathematical *phrase* built from a term or terms.

The relational operators are mathematical *verbs* that join expressions.

**EQUATION**

An equation is a mathematical *sentence* that equates two expressions; e.g., $1 + 1 = 2$

An inequality is a mathematical sentence that relates unequal expressions; e.g., $1 + 1 > 1$

```
\[
\begin{array}{ccc}
1 & + & 1 \\
\hline
\end{array}
\]
```

**SENTENCE**

```
<table>
<thead>
<tr>
<th>Phrase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word</td>
</tr>
<tr>
<td>John</td>
</tr>
<tr>
<td>Verb</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
```

**EQUATION**

```
<table>
<thead>
<tr>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Operator</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
```
Term: CV<sup>E</sup>MD

A term is a mathematical word.

English has different types of words: nouns, pronouns, etc. Similarly, math has different types of terms. A term can include any or all of the following components:

- **Constants**
  Numbers that do not vary; e.g., 100 is the number of cents in a dollar.

- **Variables**
  Letters that represent numbers that can vary; e.g., N is the number of cents in your penny jar.

- **Exponents**
  Powers assigned to constants or variables; e.g., 2<sup>3</sup>, x<sup>4</sup>.

- **Multiplication**
  Multiplied components; e.g., 3x<sup>2</sup>y.

- **Division**
  Divided components; e.g., y<sup>3</sup>/5.

**BrainAid**

Acronym: CV<sup>E</sup>MD

Acrostic: CardioVascular Expert—Medical Doctor

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### Term Families

Imagine that a term is like a person. As each person is a unique blend of body parts, each term is a unique blend of math components. A person belongs to a family whose power is determined by its wealth and social standing. A term belongs to a family whose power is determined by its exponent.

<table>
<thead>
<tr>
<th>Power</th>
<th>Family</th>
<th>Visual</th>
<th>BrainAid</th>
</tr>
</thead>
<tbody>
<tr>
<td>X&lt;sup&gt;0&lt;/sup&gt;</td>
<td>Constant Term Family</td>
<td>1</td>
<td>I’m not very strong.</td>
</tr>
<tr>
<td>(equals 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X&lt;sup&gt;1&lt;/sup&gt;</td>
<td>Line Term Family</td>
<td>x</td>
<td>I’m strong.</td>
</tr>
<tr>
<td>(equals x)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X&lt;sup&gt;2&lt;/sup&gt;</td>
<td>Square Term Family</td>
<td>x x&lt;sup&gt;2&lt;/sup&gt;</td>
<td>I’m very strong.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X&lt;sup&gt;3&lt;/sup&gt;</td>
<td>Cube Term Family</td>
<td>x x&lt;sup&gt;3&lt;/sup&gt;</td>
<td>I’m extremely strong.</td>
</tr>
</tbody>
</table>

**Term Operators**

Since ‘x’ is often used as a variable, avoid using it to show multiplication in algebra.

Instead use a dot, place items next to each other, or use parentheses:

- a • b
- ab
- (a)(b)
- a(b+c)

Use fraction lines for division:

- a/b or a/b

---

Expression: Mono or Poly

An expression is a mathematical phrase built from a term or terms.

Expressions are classified by how many terms they contain.

<table>
<thead>
<tr>
<th>EXPRESSION</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomial (moh-NOH-mee-ul)</td>
<td>$x^2$</td>
</tr>
<tr>
<td>Binomial (bii-NOH-mee-ul)</td>
<td>$x^2 + x$</td>
</tr>
<tr>
<td>Trinomial (trii-NOH-mee-ul)</td>
<td>$x^2 + x + 1$</td>
</tr>
</tbody>
</table>

- Nominal means *name*, or in this case: *term*.
- Mono means *one*. A monomial has one term.
- Bi means *two*. A binomial has two terms.
- Tri means *three*. A trinomial has three terms.
- Poly means *many*. Polynomial (paw-lee-NOH-mee-ul) is the overall word for monomials, binomials, trinomials, and other expressions with whole number exponents (i.e., not negative like $x^{-2}$ or fractional like $x^{1/2}$).

Coefficient Coworkers

Coefficients [coh-ee-FISH-untz] are constants coupled with variables.

Coefficients can be numbers, or letters that represent numbers.

Coefficient vs. Variable Letter Choices

Coefficient letters are typically taken from the *beginning* of the alphabet (e.g., a, b, c). Although they are placeholders, coefficient letters represent constants, *not* variables. To avoid confusion, variable letters are typically taken from the *end* of the alphabet (e.g., x, y, z).

Note
The $2^{nd}$ and $3^{rd}$ terms contain $x^1$ and $x^0$ as in

$4x^2 + 3x^1 + 2x^0$

But $x^1 = x$, so the exponent is omitted, and $x^0 = 1$, so there’s no need to show it.
Equation: Balancing Act
An equation is a mathematical sentence that equates two expressions.

An equation is like a balance scale that must have equal weight (expressions) on both sides to be balanced.

<table>
<thead>
<tr>
<th>Start with an empty scale in a balanced condition (indicated by the = sign).</th>
<th>Add a weight to one side to unbalance the scale (indicated by the ≠ sign).</th>
<th>Add an equal weight to the other side to rebalance the scale (indicated by the = sign).</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Balance Scale" /></td>
<td><img src="image2" alt="Unbalanced Scale" /></td>
<td><img src="image3" alt="Rebalanced Scale" /></td>
</tr>
</tbody>
</table>

Golden Rule of Equations
Whatever you do to one side, do to the other side.

PROPERTY OF EQUALITY
If \( a = b \), then
\[
\begin{align*}
\text{a + c} & = \text{b + c} \\
\text{a – c} & = \text{b – c} \\
\text{ac} & = \text{bc} \\
\text{a/c} & = \text{b/c}
\end{align*}
\]
You can add the same amount to both sides. You can subtract the same amount from both sides. You can multiply both sides by the same amount. You can divide both sides by the same amount.

To keep the scale or equation balanced, whatever you do to one side, you must do to the other side.
Algebra: Science of Equations

Algebra is the branch of mathematics that uses equations to join expressions. Algebra [AL-jeh-bruh] comes from Al Jabr, which is Arabic for “bringing together.”

Most people are comfortable with arithmetic. So why do they panic when it comes to algebra? In a word: variables. How strange that letters, which seem so natural and non-threatening when used for words, become frightening when used with numbers. To be successful with algebra, you must make friends with variables.

Variable = Box

A variable is a letter used as a placeholder for a number that can vary.

Variables are sometimes referred to as unknowns, since they represent unknown numbers. Variables are also known as literal numbers. In this case, literal means “letter.”

If we knew all the numbers in a problem beforehand, there would be no need for variables. But in real-life situations, there’s usually something we’re trying to discover. Variable placeholders allow us to manipulate an equation until we discover the unknown numbers. Imagine that variables are magic boxes that can hold any number—positive, negative, small, or large. Like a genie fitting into a bottle, even a large number can be put into a box without changing its size.

Imagine painting letters on variable boxes so we can identify them by name. The most common letter used is x, but we can use any letter, upper or lowercase.

<table>
<thead>
<tr>
<th>Box</th>
<th>Letter</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td></td>
</tr>
<tr>
<td>z</td>
<td></td>
</tr>
</tbody>
</table>

Variable letters are equal to whatever is in their boxes.

\[x = 2\]
\[y = -4\]
\[z = 6\]
Goal of Algebra: What’s in the Box?

Al Jabr says: What’s in the box?

Hi, I'm Al Jabr. My name is Arabic for "bringing together," and that's what algebra does. It brings together something we don't know (unknown) with something we do know (given). Algebra uses the given value to find the unknown value.

My body is shaped like a scale. When my arms are balanced, I have a big smile on my face!

I want to find out how many gold bars are in this treasure box. It's rusted shut and sure is heavy!

One gold bar helps, but isn’t quite enough to balance me out. Let’s try one more.

That's much better. I'm balanced now, so the treasure box must contain two gold bars!

Al Jabr cleverly adjusted his arms to counter the weight of the box, so the only thing he was measuring for was the weight of what was inside the box. Using algebraic [al-jeh-BRAA-ik] language, we can state the problem this way: How many gold bars (unknown) are in the treasure box, given that its contents are balanced by two gold bars? The obvious answer is two gold bars.

Al Jabr was right! When I opened the box, I found two gold bars inside!
Isolating the Variable: Garbo Rule

Greta Garbo says: *I vant to be alone!*

Greta Garbo was a movie star in the 1930s who came to shun publicity. She once complained in her foreign accent: *I vant to be alone!*

We’ll use Ms. Garbo’s famous lament as a BrainAid, because the variable box also “wants to be alone.” Our goal is to get the box alone on one side of the scale, so that it’s contents are revealed on the opposite side.

Isolating the Variable: Clearly Opposite

To isolate the variable, clear everything away from it with an opposite (aka reciprocal) operation.

- If a term is *added* to the variable side, clear it by *subtracting* it from both sides.
- If a term is *subtracted* from the variable side, clear it by *adding* it to both sides.
- If the variable is *multiplied* by a coefficient, clear it by *dividing* both sides by the coefficient.
- If the variable is *divided* by a coefficient, clear it by *multiplying* both sides by the coefficient.

Evaluating an Expression: Plug & Chug

To evaluate means to “find the value of” an expression given the value/s of its variable/s.

1. **Plug** in (substitute) the given value/s for the variable/s.
2. **Chug** ahead and perform the operation.

Example: If \( x = 2 \), what is the value of \( 3x + 4 \)?

\[
3x + 4 = 3(2) + 4 = 6 + 4 = 10
\]
**Proportionality**

In equations, proportionality affects how changing one item affects another.

### Proportional

Proportional [proh-POR-shun-ul] items increase (or decrease) *together* to keep the equation balanced.

<table>
<thead>
<tr>
<th>Opposite Sides</th>
<th>Same Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>As C increases, A increases.</td>
<td>As C increases, B increases.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operation</th>
<th>Example</th>
<th>Operation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>$A = B + C$</td>
<td>A</td>
<td>$A = B + C$</td>
</tr>
<tr>
<td></td>
<td>$4 = 2 + 2$</td>
<td>$A \neq B + C$</td>
<td>$4 \neq 2 + 3$</td>
</tr>
<tr>
<td></td>
<td>$5 = 2 + 3$</td>
<td>$A \neq B + C$</td>
<td>$4 \neq 2 + 3$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$A = BC$</td>
<td>$A \neq BC$</td>
<td>$2 \neq 4 \cdot 3$</td>
</tr>
<tr>
<td></td>
<td>$4 = 2 \cdot 2$</td>
<td>$A \neq BC$</td>
<td>$2 \neq 4 / 3$</td>
</tr>
<tr>
<td>Subtraction</td>
<td>$A = B - C$</td>
<td>$A \neq B - C$</td>
<td>$2 \neq 4 / 3$</td>
</tr>
<tr>
<td></td>
<td>$2 = 4 - 2$</td>
<td>$2 \neq 4 / 3$</td>
<td>$2 = 6 / 3$</td>
</tr>
<tr>
<td>Division</td>
<td>$A = \frac{B}{C}$</td>
<td>$A \neq \frac{B}{C}$</td>
<td>$2 \neq 4 / 3$</td>
</tr>
<tr>
<td></td>
<td>$2 = 4 / 2$</td>
<td>$2 \neq 4 / 3$</td>
<td>$2 = 6 / 3$</td>
</tr>
</tbody>
</table>

### Inversely Proportional

Inversely Proportional items increase (or decrease) *oppositely* to keep the equation balanced.

<table>
<thead>
<tr>
<th>Opposite Sides</th>
<th>Same Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>As C increases, A decreases.</td>
<td>As C increases, B decreases.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operation</th>
<th>Example</th>
<th>Operation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction</td>
<td>$A = B - C$</td>
<td>$A \neq B - C$</td>
<td>$2 \neq 4 - 3$</td>
</tr>
<tr>
<td></td>
<td>$2 = 4 - 2$</td>
<td>$2 \neq 4 - 3$</td>
<td>$2 = 6 / 3$</td>
</tr>
<tr>
<td>Division</td>
<td>$A = \frac{B}{C}$</td>
<td>$A \neq \frac{B}{C}$</td>
<td>$2 \neq 4 / 3$</td>
</tr>
<tr>
<td></td>
<td>$2 = 4 / 2$</td>
<td>$2 \neq 4 / 3$</td>
<td>$2 = 6 / 3$</td>
</tr>
<tr>
<td>Addition</td>
<td>$A = B + C$</td>
<td>$A \neq B + C$</td>
<td>$1 \neq 4 - 3$</td>
</tr>
<tr>
<td></td>
<td>$4 = 2 + 2$</td>
<td>$4 \neq 2 + 3$</td>
<td>$2 = 4 / 2$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$A = BC$</td>
<td>$A \neq BC$</td>
<td>$4 \neq 4 \cdot 2$</td>
</tr>
<tr>
<td></td>
<td>$4 = 4 \cdot 1$</td>
<td>$4 \neq 4 \cdot 2$</td>
<td>$4 = 2 \cdot 2$</td>
</tr>
<tr>
<td></td>
<td>$4 = 2 \cdot 2$</td>
<td>$4 \neq 4 \cdot 2$</td>
<td>$4 = 2 \cdot 2$</td>
</tr>
</tbody>
</table>
Relation: Pairing Up
A relation is a collection of ordered pairs.

Ordered Pair: (Boy, Girl)
An Ordered Pair is made of two numbers written inside parentheses in this order: (Domain, Range).

<table>
<thead>
<tr>
<th>DOMAIN</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first numbers of a collection of ordered pairs make up the Domain of the relation.</td>
<td>The second numbers of a collection of ordered pairs make up the Range of the relation.</td>
</tr>
<tr>
<td>Domain Numbers = Independent Boys</td>
<td>Range Numbers = Dependent Girls</td>
</tr>
<tr>
<td>Boys enter ordered pairs first, alone and independent (1, ) (2, ).</td>
<td>Girls enter ordered pairs second, dependent upon which boy is there (1,3) (2,4).</td>
</tr>
</tbody>
</table>

BrainAid: (Domain, Range) is in alphabetical order; i.e., D comes before R. It is also in the same order as the musical scale (Do, Re).

Types of Relations: Dating
Imagine boys and girls forming relations for dating.

- One-to-Many: (1,3)(1,4)
  One boy dates many girls.

- Many-to-Many: (1,3)(1,4)(2,3)(2,4)
  Each boy dates many girls.

- Many-to-One: (1,3)(2,3)
  Many boys date one girl.

- One-to-One: (1,3)(2,4)
  Each boy dates only one girl.
Cartesian Coordinates: \((x, y)\)

Cartesian coordinates [car-TEE-zhun koh-OR-di-nutz] are ordered pairs represented by \((x, y)\) displayed on a two-dimensional graph. The word ‘Cartesian’ comes from Rene Descartes [reh-NAA daa-KART], the 17\textsuperscript{th} century French philosopher/mathematician who conceived the system.

**Axes**

Two number lines, called axes [AX-eez], intersect at a point called the origin [OR-ih-jin], which corresponds to the ordered pair \((0,0)\).

**Axes**

- **X-Axis = Across**
- **Y-Axis = High**

**Quadrants**

The x-axis [AX-iss] and the y-axis create four quadrants [KWAW-druntz] or quarters.

**BrainAid**

Imagine the x-axis divides ground from sky.
Imagine the y-axis divides night from day.
- Positive is bright and warm.
- Negative is dark and cold.

- **I.** Upper right: Day sky \((+x, +y)\)
- **II.** Upper left: Night sky \((-x, +y)\)
- **III.** Lower left: Cold ground \((-x, -y)\)
- **IV.** Lower right: Warm ground \((+x, -y)\)

**Coordinates**

**X-Coordinate:** The variable \(x\) independently goes right or left across the Domain (think region or territory).

**Y-Coordinate:** The variable \(y\), whose value is dependent upon \(x\), goes high up/down the Range (think mountain).

**BrainAid:** Imagine a king hiking independently across his domain. His elevation in the mountain range is dependent on his position in his domain.

**The King’s travels:** At \((-3,-1)\) he is 3 left and 1 down from the \((0,0)\) origin, which is the center of his kingdom.
At \((-1,1)\) he is 1 left and 1 up. At \((1,-2)\) he is 1 right and 2 down. At \((5,3)\) he is 5 right and 3 up from the origin.
Plotting Ordered Pairs: x across, y high

To “plot” an ordered pair means to find then draw and label a point on a Cartesian-Coordinate graph.

<table>
<thead>
<tr>
<th>To plot (x,y) stand at the origin (0,0) then:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Think x is across!</strong></td>
</tr>
<tr>
<td>If x is positive, step right.</td>
</tr>
<tr>
<td>If x is negative, step left.</td>
</tr>
<tr>
<td><strong>Think y is high!</strong></td>
</tr>
<tr>
<td>If y is positive, leap up.</td>
</tr>
<tr>
<td>If y is negative, dig down.</td>
</tr>
<tr>
<td><strong>Make A Point!</strong></td>
</tr>
<tr>
<td>Draw a dot and label it with the ordered pair.</td>
</tr>
</tbody>
</table>

Plotting Relations: All over the map

Compare each plot below to the type of relation shown on page 20.

One x to Many y

Many x to Many y

Many x to One y

One x to One y

Tip
When plotting coordinates, it’s easier and more accurate to use graph paper with preprinted grids.

3-D!
FYI: With the addition of a Z-axis running front-to-back, coordinates can be plotted in three dimensions. (x, y, z).
Function: Fun at the Junction

A function is a relation where each domain value \( x \) has only one range value \( y \).
The value of \( y \) is a function of (i.e., depends upon) the value of \( x \).

**Discrete vs. Continuous: No line vs. line**

**Discrete Function** plots consist of separate points that *cannot* be connected by a line.
Discrete means separate or distinct. Splitting discrete items (e.g., people, objects, places) into smaller pieces doesn’t make sense.
Example: The total cost (range) of toys that cost $1 each is a function of the number of toys (domain) purchased.

**Continuous Function** plots consist of an unbroken line of points.
Continuous items (e.g., time, speed, distance, temperature) can be reasonably split into smaller pieces.
Example: The distance (range) a vehicle going 1 mile/minute travels is a function of the time (domain) it has traveled.

**Vertical Line Test: One \( y \) per \( x \)**

If a vertical line can be drawn through two or more points of a relation, it’s *not* a function.

Explanation: Since, by definition, each domain value in a function can have only one range value, functions are limited to Many-to-One or One-to-One relations (p.20).
Standard Function Layout: \( y = x \)

Range variable = Domain expression

\[ y = x \text{ expression} \]

Isolating the \( y \) variable on the left makes it easy to see that it’s a function of (i.e., depends upon) the \( x \) expression on the right.

If \( x \) changes, \( y \) changes.

Example: \( y = x + 1 \)
- If \( x = 2 \), then \( y = (2) + 1 = 3 \)
- If \( x = 3 \), then \( y = (3) + 1 = 4 \)

\[ f(x) = x \text{ expression} \]

- or -

Using \( f(x) \) in place of \( y \) has the advantage of showing the \( x \) value that produces the range value, e.g.,

\[ f(x) = x + 1 \]
\[ f(2) = 2 + 1 \]
\[ f(2) = 3 \]

\( f(x) \), pronounced \( f \) of \( x \), means “function of \( x \),” not “\( f \) times \( x \).”

### Function Families

Functions, like the terms they contain, can be classified into families based on the power of their exponents (see Term Families p.13). Each function family has a different shape when graphed.

The arrows on the ends of lines and curves indicate that they continue forever—to infinity!

<table>
<thead>
<tr>
<th>Constant [KAWN-stunt]</th>
<th>Linear [L.H-nee-ur]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function Family</strong></td>
<td><strong>Function Family</strong></td>
</tr>
<tr>
<td>Flat Line</td>
<td>Sloped Line</td>
</tr>
<tr>
<td>( y = x^0 )</td>
<td>( y = x^1 )</td>
</tr>
<tr>
<td>( -2 ) ( 1 )</td>
<td>( -2 ) ( -2 )</td>
</tr>
<tr>
<td>( -1 ) ( 1 )</td>
<td>( -1 ) ( -1 )</td>
</tr>
<tr>
<td>( 0 ) ( 1 )</td>
<td>( 0 ) ( 0 )</td>
</tr>
<tr>
<td>( 1 ) ( 1 )</td>
<td>( 1 ) ( 1 )</td>
</tr>
<tr>
<td>( 2 ) ( 1 )</td>
<td>( 2 ) ( 1 )</td>
</tr>
</tbody>
</table>

### Quadratic [kwaw-DRA-tik]

Parabola [puh-RA-boh-luh]

<table>
<thead>
<tr>
<th>( y = x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -2 ) ( 4 )</td>
</tr>
<tr>
<td>( -1 ) ( 1 )</td>
</tr>
<tr>
<td>( 0 ) ( 0 )</td>
</tr>
<tr>
<td>( 1 ) ( 1 )</td>
</tr>
<tr>
<td>( 2 ) ( 4 )</td>
</tr>
</tbody>
</table>

**BrainAid**

A parabola is bowl-shaped.

### Cubic [KYU-bik]

Curve

<table>
<thead>
<tr>
<th>( y = x^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -2 ) ( -8 )</td>
</tr>
<tr>
<td>( -1 ) ( -1 )</td>
</tr>
<tr>
<td>( 0 ) ( 0 )</td>
</tr>
<tr>
<td>( 1 ) ( 1 )</td>
</tr>
<tr>
<td>( 2 ) ( 8 )</td>
</tr>
</tbody>
</table>

**BrainAid**

A cubic curve waves “hi!”
Operations

One Equation, One Unknown

The simplest algebra problems have One Equation with One first-power ($x^1$) Unknown. For short, we’ll call these 1EqUnk [ek-unk] problems.

**1EqUnk Added Term ($x + 2 = 4$)**

**Goal:** What’s in the box?

**Garbo Rule:** Get the box alone.

**Clearly Opposite:** Subtract the added amount from each side.

To avoid errors, keep terms and operators lined up. Circle the solution.

**Important!** Plug the solution back into the original equation to make sure it’s correct.

**Check**

$x + 2 = 4$

$2 + 2 = 4$

These cancel

$4 = 4 \checkmark$

**Tips**

Steal a 2-high pile from each side.

Your turn: Solve for $x$ by subtracting.

<table>
<thead>
<tr>
<th>Solve</th>
<th>Check</th>
<th>Solve</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 3 = 4$</td>
<td>$x + 3 = 4$</td>
<td>$x + 2 = 5$</td>
<td>$x + 2 = 5$</td>
</tr>
<tr>
<td>$x + 5 = 9$</td>
<td>$x + 5 = 9$</td>
<td>$x + 6 = 15$</td>
<td>$x + 6 = 15$</td>
</tr>
</tbody>
</table>
1EqUnk Subtracted Term (x – 2 = 4)

**Goal:** What’s in the box?
**Garbo Rule:** Get the box alone.
**Clearly Opposite:** Add the subtracted amount to both sides.

**Importantly!**
Plug the solution back into the original equation to make sure it’s correct.

**Check**
\[
x - 2 = 4 \\
6 - 2 = 4 \\
4 = 4 \quad \sqrt{\ }
\]

**Tips**
To avoid errors, keep terms and operators lined up.
Circle the solution.

---

**Your turn:** Solve for x by adding.

<table>
<thead>
<tr>
<th>Solve</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>x – 3 = 4</td>
<td>x - 3 = 4</td>
</tr>
<tr>
<td>x – 4 = 5</td>
<td>x – 4 = 5</td>
</tr>
<tr>
<td>x – 5 = 9</td>
<td>x – 5 = 9</td>
</tr>
<tr>
<td>x – 6 = 15</td>
<td>x – 6 = 15</td>
</tr>
</tbody>
</table>

1EqUnk Multiplied Variable (2x = 4)

Goal: What’s in the box?
Garbo Rule: Get the box alone.
Clearly Opposite: Divide each side by the multiplied amount.

Solve:

\[ 2x = 4 \]

Divide each side by 2:

\[ \frac{2x}{2} = \frac{4}{2} \]

These dissolve:

\[ x = 2 \]

Important!
Plug the solution back into the original equation to make sure it’s correct.

Check:

\[ 2x = 4 \]
\[ 2(2) = 4 \]
\[ 4 = 4 \]

Tips
To avoid errors, keep terms and operators lined up.
Circle the solution.

Your turn: Solve for x by dividing.

<table>
<thead>
<tr>
<th>Solve</th>
<th>Check</th>
<th>Solve</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x = 6</td>
<td>2x = 6</td>
<td>3x = 6</td>
<td>3x = 6</td>
</tr>
<tr>
<td>4x = 20</td>
<td>4x = 20</td>
<td>5x = 20</td>
<td>5x = 20</td>
</tr>
</tbody>
</table>
1EqUnk Divided Variable \((x/2 = 4)\)

**Goal:** What’s in the box?

**Garbo Rule:** Get the box alone.

**Clearly Opposite:** Multiply each side by the divided amount.

---

**Solve**

\[
\frac{x}{2} = 4
\]

**Check**

\[
\frac{x}{2} = 4
\]

---

**Tips**

To avoid errors, keep terms and operators lined up.

Circle the solution.

---

**Your turn:** Solve for \(x\) by multiplying.

<table>
<thead>
<tr>
<th>Solve</th>
<th>Check</th>
<th>Solve</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x = 6) (\frac{2}{2})</td>
<td>(x = 6) (\frac{2}{2})</td>
<td>(x = 1) (\frac{3}{3})</td>
<td>(x = 1) (\frac{3}{3})</td>
</tr>
<tr>
<td>(x = 2) (\frac{4}{4})</td>
<td>(x = 2) (\frac{4}{4})</td>
<td>(x = 3) (\frac{5}{5})</td>
<td>(x = 3) (\frac{5}{5})</td>
</tr>
</tbody>
</table>

---

**Important!**

Plug the solution back into the original equation to make sure it’s correct.

\[
\frac{x}{2} = 4
\]

\[
\frac{8}{2} = 4
\]

\[
4 = 4
\]
Multiple Operations: Clear As Mud

When an equation contains multiple operators, it may not be clear what you should do first. In fact, it’s as clear as mud!

1st Clear away any term/s Added or Subtracted to the variable.
2nd Clear away any coefficient/s from a Multiplied or Divided variable.

BrainAid: It’s as clear A/S M/D. Added/Subtracted; Multiplied/Divided.

<table>
<thead>
<tr>
<th>Solve</th>
<th>CLEAR</th>
<th>Solve</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3x + 7 = 13)</td>
<td>(3x + 1 = 13)</td>
<td>(x - 5 = 1)</td>
</tr>
<tr>
<td>Clear Added term by subtracting 7 from both sides.</td>
<td>AS</td>
<td>Clear Subtracted term by adding 5 to both sides.</td>
</tr>
<tr>
<td>(\frac{3x}{3} + \frac{7}{3} = 13)</td>
<td>(\frac{x}{2} - \frac{5}{2} = 1)</td>
<td>Clear Divided variable by multiplying both sides by 2.</td>
</tr>
<tr>
<td>(x = 6)</td>
<td>(x = 6(2))</td>
<td>(x = 12)</td>
</tr>
<tr>
<td>Check</td>
<td>Check</td>
<td>Check</td>
</tr>
<tr>
<td>(3(2) + 7 = 13)</td>
<td>(\frac{12}{2} - \frac{5}{2} = 1)</td>
<td>(6 - 5 = 1)</td>
</tr>
<tr>
<td>(6 + 7 = 13)</td>
<td>(6 - 5 = 1)</td>
<td>(1 = 1)</td>
</tr>
<tr>
<td>(13 = 13)</td>
<td></td>
<td>(1 = 1)</td>
</tr>
</tbody>
</table>

Your turn: Solve using the Clear-As-Mud procedure.

<table>
<thead>
<tr>
<th>Solve</th>
<th>(2x - 3 = 7)</th>
<th>Solve</th>
<th>(\frac{x}{3} + 1 = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3x + 1 = 13)</td>
<td>(\frac{x}{2} - \frac{5}{2} = 1)</td>
<td>(\frac{x}{3} + \frac{1}{3} = \frac{2}{3})</td>
<td>(\frac{x}{2} - \frac{5}{2} = 1)</td>
</tr>
<tr>
<td>Clear Added term by subtracting 7 from both sides.</td>
<td>AS</td>
<td>Clear Subtracted term by adding 5 to both sides.</td>
<td>AS</td>
</tr>
<tr>
<td>(\frac{2x}{3} - \frac{3}{3} = 7)</td>
<td>(\frac{x}{2} - \frac{5}{2} = 1)</td>
<td>(\frac{x}{3} + \frac{1}{3} = \frac{2}{3})</td>
<td>(\frac{x}{2} - \frac{5}{2} = 1)</td>
</tr>
<tr>
<td>(\left(\frac{2}{3}\right)x = 6)</td>
<td>(\frac{x}{2} + \frac{1}{2} = 2)</td>
<td>(\left(\frac{2}{3}\right)x = \frac{2}{3})</td>
<td>(\left(\frac{2}{3}\right)x = \frac{2}{3})</td>
</tr>
<tr>
<td>(\frac{x}{3} = 2)</td>
<td>(\frac{x}{2} = 3)</td>
<td>(\frac{x}{3} = 1)</td>
<td>(\frac{x}{2} = 3)</td>
</tr>
<tr>
<td>Check</td>
<td>Check</td>
<td>Check</td>
<td>Check</td>
</tr>
<tr>
<td>(\left(\frac{2}{3}\right)2 + 1 = 2)</td>
<td>(\frac{12}{2} - \frac{5}{2} = 1)</td>
<td>(6 - 5 = 1)</td>
<td>(1 = 1)</td>
</tr>
<tr>
<td>(\left(\frac{2}{3}\right)2 + 1 = 2)</td>
<td>(6 - 5 = 1)</td>
<td>(1 = 1)</td>
<td>(1 = 1)</td>
</tr>
</tbody>
</table>
Simplifying Terms

Multiple Terms: Family Reunion

In expressions with multiple terms, combine *like* terms.

*Like* (aka similar) terms have the same variable/s raised to the same power/s (Term Families p.13).

Draw a box around each term, *including its sign.*

Draw lines from like terms, and combine them into single terms.

Place the highest power term on the left and proceed in descending order: $x^2, x^1, x^0$.

**BrainAid:** Imagine like terms combining together at family reunions.

Each family’s value is a mix of the positive and negative personalities (coefficients) of its members.

**Your turn:** Simplify the expressions by holding Family Reunions.

<table>
<thead>
<tr>
<th>Simplify</th>
<th>Simplify</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x - 6 + 3x + 1$</td>
<td>$-4x + 6 + 2x + 3 + x^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simplify</th>
<th>Simplify</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x + 7 - 4x^2 - 6 + 3x + x^2$</td>
<td>$5x + 2 - 7x^2 - 6x + 3x^2 + 1$</td>
</tr>
</tbody>
</table>
Separated Terms: Take Sides
If like terms are on opposite sides of the equal sign, move them to the same side and combine.

<table>
<thead>
<tr>
<th>Simplify</th>
<th>Your turn: Simplify</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 3x + 2 = 4 - 2x ]</td>
<td>[ 4x - 3 = 5 + 2x ]</td>
</tr>
<tr>
<td>[ \text{Move variable left.} ] [ \frac{3x}{2} + \frac{2}{2} = 4 ] [ \frac{5x}{2} = 2 ]</td>
<td>[ \text{Which Side?} ] It’s traditional to move variables to the left side, but not mandatory. For example, ( x = 1 ) is the same as ( 1 = x ) It’s preferable to move variables to the side that results in a positive coefficient. [ \frac{1}{1} = 2x ] [ -x ] [ \frac{1}{x} ]</td>
</tr>
</tbody>
</table>
| \[ \text{Move constant right.} \] \[ \frac{3x}{2} + \frac{2}{2} = 4 \] \[ \frac{5x}{2} = 2 \] | \[ \text{Distributed Terms: Fair to All} \] If terms in parentheses are multiplied by an outer term, distribute equally to every inner term. Take special care to distribute negatives correctly.

<table>
<thead>
<tr>
<th>Distribute to each inner term</th>
<th>Your turn:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 4(x - 2) = 20 ] [ 4x - 8 = 20 ]</td>
<td>[ 5(-x + 2) = 25 ]</td>
</tr>
<tr>
<td>[ \text{Distribute the negative number} ] [ -4(x - 2) = 20 ] [ -4x + 8 = 20 ]</td>
<td>[ \text{Distribute the negative number} ] [ -5(-x + 2) = 25 ]</td>
</tr>
<tr>
<td>[ \text{Distribute the minus sign} ] [ -(x - 2) = 20 ] [ -x + 2 = 20 ]</td>
<td>[ \text{Distribute the minus sign} ] [ -(-x + 2) = 25 ]</td>
</tr>
<tr>
<td></td>
<td>This is equivalent to multiplying by (-1).</td>
</tr>
</tbody>
</table>
Simplifying Coefficients: Throw Mud

To simplify coefficients, it may help to “throw a little mud” at them first.

Clear Denominators: Magnify by LCM

To clear constants or variables that appear in denominators, *throw mud* to multiply all terms by the LCM (p.11). This is the same as multiplying each side by the same amount, so the equation remains equal. If only one term has a denominator, it’s the LCM, so multiply all terms by it.

Reduce Coefficients: Dissolve with GCF

To reduce coefficients, *throw mud* by dividing the GCF (p.10) into each term. If the variable coefficient is negative, divide by a negative GCF.
Fractional Terms

Clearing Equated Fractions: Shoot-the-Chute

When fractional expressions are set equal to each other, cross multiply to clear their fractions and isolate the variable.

\[
\frac{3}{4} = \frac{2}{x}
\]

**BrainAid:** Shoot-the-Chute is an amusement park ride that has a chute or slide. Imagine that the equal sign between expressions is a chute that tilts so numbers and variables can “shoot” up or down through it.

**Solve**

<table>
<thead>
<tr>
<th>Solve</th>
<th>Why It Works</th>
</tr>
</thead>
</table>
| \[
\frac{3}{4} = \frac{2}{x}
\] | \[
(x)\frac{3}{4} = \frac{2(x)}{x}
\] |
| Tilt chute. Shoot 3 down. Shoot x up. | \[
\frac{3x}{4} = 2
\] |
| Tilt chute. Shoot 4 up. $4 \times 2 = 8$ | Divide both sides by 3 |
| Un-tilt chute. | \[
\frac{x}{4} = \frac{2}{3}
\] |
| | Multiply both sides by 4 |
| | \[
\frac{4x}{4} = \frac{2(4)}{3}
\] |
| | \[
x = \frac{8}{3}
\] |

**Your turn:** Shoot-the-Chute to clear the fractions and isolate the variable.

\[
\frac{3}{5} = \frac{1}{x}
\]

\[
\frac{2x}{3} = \frac{3}{7}
\]

\[
\frac{5}{2} = \frac{4}{3x}
\]
Combining Fractions: Spotlighting!

As an alternative to clearing denominators by magnifying with the LCM (p.32), use the spotlighting technique to create equivalent fractions. (See Max Learning’s Fraction Fun: Xdm/Sh!). If the original denominators are not prime numbers, factor them and “crush” any common factors before spotlighting.

**Spotlight**

\[
\frac{x}{2} + \frac{x}{3} = 4
\]

\[
\frac{3x}{6} + \frac{x}{2} + \frac{2x}{6} = 4
\]

\[
\frac{5x}{6} = 4
\]

**Crush & Spotlight**

\[
\frac{x}{4} + \frac{x}{6} = 10
\]

\[
\frac{3x}{12} + \frac{x}{2} + \frac{2x}{12} = 10
\]

\[
\frac{5x}{12} = 10
\]

Tip: To isolate x from here, use Shoot-the-Chute (p.33).

**Your turn:** Spotlight to combine fractions.

\[
\frac{x}{5} + \frac{x}{2} = 3
\]

\[
\frac{2x}{3} - \frac{x}{2} = 8
\]

**Your turn:** Crush & spotlight to combine fractions.

\[
\frac{2x}{3} + \frac{x}{6} = 7
\]

\[
\frac{3x}{4} - \frac{5x}{8} = 9
\]
Two Equations, Two Unknowns

Some algebra problems involve Two Equations with Two first-power \((x^1 \& y^1)\) Unknowns, aka *simultaneous equations* or a *system of equations*. Their solution, if one exists, is the ordered pair (p.20) that satisfies both equations. For short, we’ll call these 2EqUnk [ek-unk] problems.

**Dilemma**
If you have two equations each with two boxes (variables) on a scale, it’s not obvious how to isolate either box to see what it contains.

**Remedy**
Use Elimination (p.36) or Substitution (p.40) to transform the two equations into one equation with one unknown (1EqUnk p.25). Solve it, and use its solution to find the value of the second unknown variable.

### Possible 2EqUnk Outcomes
2EqUnk equations belong to the Linear Function Family (p.24), so each equation will graph as a sloped line.

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Description</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intersecting Lines</strong></td>
<td>The solution is the point where the two lines cross</td>
<td><img src="image" alt="Diagram of intersecting lines" /></td>
</tr>
<tr>
<td><strong>Identical (Collinear) Lines</strong></td>
<td>The solution is every point on each line.</td>
<td><img src="image" alt="Diagram of identical lines" /></td>
</tr>
<tr>
<td><strong>Parallel [PAIR-uh-lel] Lines</strong></td>
<td>There is no solution, as parallel lines never touch.</td>
<td><img src="image" alt="Diagram of parallel lines" /></td>
</tr>
</tbody>
</table>

**Spelling Tip**
To spell “parallel,” imagine you have a friend named El who likes to golf. To wish him luck, you say, “I hope you par all El!”
2EqUnk Elimination: You’re outa here!

Use Elimination when both equations are in \( ax + by = c \) form.

1. Combine the 2EqUnks so as to eliminate *either* variable (pick the easier one).
2. Solve the resulting 1EqUnk to get the value of its variable.
3. Plug that value into either original equation, and solve for the eliminated variable.
4. Check the \((x, y)\) solution in *both* original equations.

**Add to Eliminate**

*Add* equations that have matching, *oppositely*-signed variable terms.

\[
\begin{align*}
\text{Solve} & \quad \begin{align*}
x + y &= 2 \\
x - y &= 2
\end{align*} \\
1. \text{Add to eliminate } y & \quad \begin{align*}
x + y &= 2 \\
+ x - y &= 2 \\
\hline
2x &= 4
\end{align*} \\
2. \text{Solve 1EqUnk} & \quad \begin{align*}
\frac{1}{2}x &= 2 \\
x &= 4
\end{align*}
\end{align*}
\]

\[
\begin{align*}
\text{Solution} & \quad (x, y) = (2, 0)
\end{align*}
\]

It’s sufficient to Plug & Chug (p.18) just one equation, but always check both equations.
Subtract to Eliminate

*Subtract* equations that have matching, *same*-signed variable terms.

\[ \begin{align*}
\text{Solve} \\
x + 2y &= 4 \\
x - y &= 1
\end{align*} \]

1. Subtract to eliminate \( x \)

\[ \begin{align*}
x + 2y &= 4 \\
-x + y &= -1
\end{align*} \]

\[ 3y = 3 \]

2. Solve 1EqUnk

\[ 3y = 3 \]

\[ y = 1 \]

3. Plug & Chug

\[ x + 2y = 4 \]

\[ x + 2(1) = 4 \]

\[ x + 2 = 4 \]

\[ x = 2 \]

4. Check

\[ x + 2y = 4 \]

\[ 2 + 2(1) = 4 \]

\[ 2 + 2 = 4 \]

\[ 4 = 4 \]

Solution

\((x, y) = (2, 1)\)

It’s sufficient to Plug & Chug (p.18) just one equation, but always check both equations.
### Your turn: Add to eliminate and solve.

|-----------|--------|-----------------|----------------|---------|----------|
| \(x + 2y = 5\) | \[
x + 2y = 5 \\
+ x - 2y = 1
\] | | \[x, y = (__, __)\] | | |  
| \(x - 2y = 1\) | | | | | |

### Your turn: Subtract to eliminate and solve.

|-----------|-------------|-----------------|----------------|---------|----------|
| \(2x + 2y = 6\) | \[
2x + 2y = 6 \\
-(2x - y = 0)
\] | | \[x, y = (__, __)\] | | |  
| \(2x - y = 0\) | | | | | |
Multiply Then Eliminate

If the equations have no matching variable terms, multiply to create them.

<table>
<thead>
<tr>
<th>No matching terms</th>
<th>Multiply to eliminate y</th>
<th>No matching terms</th>
<th>Multiply to eliminate x</th>
<th>No matching terms</th>
<th>Multiply to eliminate x</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x + 2y = 4</td>
<td>3x + 2y = 4</td>
<td>2x + 3y = 5</td>
<td>2x + 3y = 5</td>
<td>3x + 4y = 7</td>
<td>3x + 4y = 7</td>
</tr>
<tr>
<td>2x - y = 3</td>
<td>2(2x - y = 3)</td>
<td>x + 2y = 2</td>
<td>-2(x + 2y = 2)</td>
<td>-2x + 3y = 1</td>
<td>3(-2x + 3y = 1)</td>
</tr>
</tbody>
</table>

Multiply to eliminate y

Eliminate

3x + 2y = 4
+ 4x - 2y = 6
7x = 10

Multiply to eliminate x

Eliminate

2x + 3y = 5
+ -2x - 4y = -4
-y = 1

Multiply to eliminate x

Eliminate

6x + 8y = 14
+ -6x + 9y = 3
17y = 17

Your turn: Multiply then eliminate.

4x + 3y = 5
2x - y = 1

Multiply to eliminate y

Eliminate

3x + 3y = 1
x + 2y = 5

Multiply to eliminate x

Eliminate

3x + 3y = 8
2x - 2y = 3

Multiply to eliminate y

Eliminate

In each case, you could choose to eliminate the opposite variable. The ultimate solution would be the same.
2EqUnk Substitution: Identical Twins

Use Substitution when one of the equations has an isolated x or y variable, or when one of the variables is relatively easy to isolate.

1. Substitute the isolated variable’s equivalent expression into the other equation.
2. Solve the resulting 1EqUnk to get the value of the other variable.
3. Plug the other variable’s value into the isolated variable’s equation and solve.
4. Check the (x, y) solution in the other equation.

Your turn: Substitute to solve.

1. Substitute
\[ x = 1 + y \]
\[ x + y = 5 \]

2. Solve 1EqUnk
\[ x + y = 5 \]
\[ (1+y) + y = 5 \]
\[ 1 + 2y = 5 \]
\[ 2y = 4 \]
\[ y = 2 \]

3. Plug & Chug
\[ x = 1 + y \]
\[ x = 1 + 2 \]
\[ x = 3 \]

Solution
\( (x, y) = (3, 2) \)

4. Check
\[ x + y = 5 \]
\[ 3 + 2 = 5 \]
\[ 5 = 5 \]

Your turn: Substitute to solve.

1. Substitute
\[ y = 2 + x \]
\[ x + y = 6 \]

2. Solve 1EqUnk

3. Plug & Chug
\[ y = 2 + x \]

Solution
\( (x, y) = (__, __) \)

4. Check
\[ x + y = 6 \]
If elimination or substitution result in variable-less equalities, the lines produced are identical (aka collinear koh-LIN-ee-ur). This occurs when one equation is a multiple of the other. EQUALITY = IDENTICAL

\[
\begin{align*}
2x - 2y &= 4 \\
x - y &= 2
\end{align*}
\]

**By Elimination**

\[
\begin{align*}
2x - 2y &= 4 \\
-2(x - y) &= -2(2) \\
2x - 2y &= 4 \\
-2x + 2y &= -4 \\
0 + 0 &= 0
\end{align*}
\]

Variable-less Equality

**By Substitution**

\[
\begin{align*}
x - y &= 2 \\
x &= 2 + y \\
2x - 2y &= 4 \\
2(2+y) - 2y &= 4 \\
4 + 2y - 2y &= 4 \\
4 &= 4
\end{align*}
\]

Variable-less Equality

If elimination or substitution result in variable-less inequalities, the lines produced are parallel. This occurs when the equations have the same coefficients but different constants. INEQUALITY = PARALLEL

**Solve**

\[
\begin{align*}
x - y &= 1 \\
x - y &= -1
\end{align*}
\]

**By Elimination**

\[
\begin{align*}
x - y &= 1 \\
-(x - y) &= -(1) \\
0 + 0 &< 2
\end{align*}
\]

Variable-less Inequality

**By Substitution**

\[
\begin{align*}
x - y &= 1 \\
x &= 1 + y \\
x - y &= -1 \\
1 + y - y &= -1 \\
1 &> -1
\end{align*}
\]

Variable-less Inequality

In some cases, it’s an all or nothing solution.

**Identical** lines share all points.

**Parallel** lines share no points.
Linear Equations

Equations involving \( x^1 \) are called first-degree or Linear [LIHN-e-ur] Equations.

When graphed, Linear Equations produce lines.

We’ll call Linear Equations LinEqs [lin-eks] for short.
- Standard form for a one-variable LinEq is \( ax + b = c \) (see 1EqUnk p.25).
- Standard form for a two-variable LinEq is \( ax + by = c \) (see 2EqUnk p.35).
- Standard form for the Linear Function is: \( f(x) = mx + b \) (see Function p.23).

Trap! The ‘b’ in \( ax + by = c \) is different from the ‘b’ in \( f(x) = mx + b \).

Slope-Intercept Form: \( y = mx + b \)

Slope-intercept form makes it easier to visualize and graph lines on Cartesian axes (p.20). The y-coefficient must be +1. The x-coefficient ‘m’ is the slope. The constant ‘b’ is the y-intercept.

We can convert a standard two-variable LinEq to slope-intercept form by isolating the y variable.

\[
\begin{align*}
\text{Convert to slope-intercept form} & \\
2x + 3y &= 9 \\
& -2x \\
\frac{3y}{3} &= -2x + 9 \\
y &= -\frac{2}{3}x + 3 \\
m &= -\frac{2}{3}; b &= 3
\end{align*}
\]

Nature of LinEqs

LinEqs represent things that occur or change at a constant rate.

Problem: Starting 1 mile from home, Tia walks at a steady rate of 1 mile per hour towards the next town. How many miles from home is she after 2 hours?

Analysis: This is a Distance = Rate \( \times \) Time (D=RT) travel problem (p.64) that fits neatly into slope-intercept form with \( y = \text{Distance} \), \( m = \text{Rate} \), \( x = \text{Time} \), and \( b = \text{starting point} \).

\[
y = mx + b \\
D = RT + b
\]

Plugging in the given values:

\[
\begin{align*}
D &= 1 \text{ mile} \times 2 \text{ hours} + 1 \text{ mile/hour} \\
D &= 2 \text{ miles} + 1 \text{ mile} \\
D &= 3 \text{ miles}
\end{align*}
\]

Solution: Starting 1 mile from home, after 2 hours of walking, Tia was 3 miles from home.

This graph shows Tia’s distance (miles) from home for the time (hours) that she walked.

The usual solution to a LinEq problem is a single point, but the line produced by the LinEq displays all possible solutions, e.g., after 3 hours of walking, Tia was 4 miles from home.
Slope: \( m = \text{mountain} \)
In the LinEq \( y = mx + b \), ‘\( m \)’ equals the slope.
The slope determines the direction and steepness of a line.

**Direction of Slope**
If ‘\( m \)’ is negative, the line slopes down from the left.
If ‘\( m \)’ is positive, the line slopes up to the right.

**Steepness of Slope**
Larger ‘\( m \)’ = Steeper slope

Arrowheads indicate that a line theoretically goes on forever in both directions.

**Slope = Rise/Run**
Slope is the ratio of a line’s rise (up/down) over its run (left/right).

**BrainAid:** Imagine steps that rise and run along the mountain slope to make it easier to climb or descend.

<table>
<thead>
<tr>
<th>( m = 1 )</th>
<th>( m = -2 )</th>
<th>( m = -2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 rise / 1 run</td>
<td>-2 rise / 1 run</td>
<td>2 rise / -1 run</td>
</tr>
</tbody>
</table>

Steps can be drawn below or above the slope line. With a negative slope, either rise or run can be negative.
Calculating Slope: \( \Delta y / \Delta x \)

\[
\frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

The delta symbol \( \Delta \) means “change in.”

\((x_2, y_2)\) and \((x_1, y_1)\) represent any two points on a line.

**Traps!**

Common Slope-Calculation Errors

- **Inconsistent subtraction direction**
  \[ \frac{y_2 - y_1}{x_1 - x_2} \]

- **Inverted coordinates**
  \[ \frac{x_2 - x_1}{y_2 - y_1} \]

- **Errors with negatives**
  \[-5 - 2 = -7 \quad \text{(should be } -3)\]

**Drop, Rotate, & Subtract**

To minimize slope-calculation errors, drop \( y \)'s down, rotate \( x \)'s around, then subtract.

\[
\begin{align*}
\text{(x}_2, y_2) & \quad \text{(x}_1, y_1) \\
y_2 & \quad y_1 \\
\frac{\Delta y}{\Delta x} & = \frac{y_2 - y_1}{x_2 - x_1} \\
& = m
\end{align*}
\]

**Your turn:** Drop, Rotate, & Subtract to find the slopes.

\begin{align*}
(3, 7) & \quad (2, 5) \\
(-3, 7) & \quad (-2, 5) \\
(-3, -7) & \quad (-2, -5)
\end{align*}

**Horizontal Line = Zero Slope**

- \( y = 3 \)
- \( y = 0 \)
- \( y = -4 \)

**Vertical Line = Undefined Slope**

- \( x = 4 \)
- \( x = 0 \)
- \( x = -4 \)

**BrainAid**

- Imagine walking on horizontal wires \[ y’urz \].
- Horizontal lines are functions (p.23).
- Paradox: The equation for the \( x \)-axis is \( y = 0 \).
**Y-intercept: \( b = \text{ball} \)**
In the LinEq \( y = mx + b \), ‘\( b \)’ equals the y-intercept. The y-intercept is the point where a line crosses the y-axis. The y-intercept occurs when \( x = 0 \).

### BrainAid:
Imagine a ball rolling down a slope being intercepted by the y-axis.

<table>
<thead>
<tr>
<th>( y = x + 2 )</th>
<th>( y = x )</th>
<th>( y = -x - 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 1 ); ( b = 2 )</td>
<td>( m = 1 ); ( b = 0 )</td>
<td>( m = -1 ); ( b = -3 )</td>
</tr>
</tbody>
</table>

### X-intercept: \( x = -\frac{b}{m} \)
The x-intercept is the point where a line crosses the x-axis. The x-intercept occurs when \( y = 0 \).

### BrainAid:
Imagine the x-axis intercepting a negative ball \((-b)\) over a mountain \((-\frac{b}{m})\).

### BrainAid:
Imagine a ball rolling down a slope being intercepted by the x-axis.
Plotting LinEqs

It takes a minimum of two points to define a line. You have several plotting options.

### Plot using y-intercept and slope

Use with slope-intercept y=mx+b form.
1. Draw the y-intercept point on the y-axis.
2. From that point, follow rise/run to the 2nd point.

**Your turn:** Plot using y-intercept and slope.

\[
y = 2x+1
\]

\[
\begin{align*}
b &= 1 \\
m &= \frac{2}{1}
\end{align*}
\]

\[
y = 3x-2
\]

\[
\begin{align*}
b &= \_\_\_ \\
m &= \_\_\_
\end{align*}
\]

### Plot using x & y intercepts

Use with standard ax+by=c form.
1. Set x=0, solve for y, plot the y-intercept.
2. Set y=0, solve for x, plot the x-intercept.

**Your turn:** Plot using x & y intercepts.

\[
-2x+y = 1
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-(\frac{1}{2})</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
-3x+y = -2
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

### Plot using x & y coordinates

Alternate for standard ax+by=c form.
1. Choose any x, solve for y, plot the point.
2. Choose any 2nd x, solve for y, plot the point.

**Your turn:** Plot using x & y coordinates.

\[
-2x+y = 1
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
-3x+y = -2
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

### Parallel & Perpendicular Line Slopes

Given two separate lines with slopes \(m_1\) and \(m_2\):
- If \(m_1 = m_2\), the lines are parallel (never touch).
- If \(m_1 \cdot m_2 = -1\), the lines are perpendicular [pur-pen-DIH-kyu-lur] (cross at 90º angles).

\[
m_1 = 1 \quad m_2 = 1
\]

\[
m_1 = 1 \quad m_2 = -1
\]
Quadratic Equations

Equations involving \( x^2 \) are called second-degree or Quadratic [kwaw-DRA-tik] Equations. When graphed, Quadratic Equations produce bowl-shaped parabolas (p.24). We’ll call Quadratic Equations QuadEqs [kwaw-deks] for short.

The Quadratic Function is \( f(x) = ax^2+bx+c \) (see Function p.23).

\[ ax^2 = \text{quadratic term, } bx = \text{linear term, } c = \text{constant term} \]

Standard Form: \( ax^2 + bx + c = 0 \)

A standard QuadEq is a special case of the Quadratic Function where \( f(x) = 0 \).

Traditionally, the 0 is moved to the right side of the equation.

A standard QuadEq is a trinomial (p.14), but it can be a binomial or monomial as follows:

If \( c=0 \): \( ax^2 + bx = 0 \)
If \( b=0 \): \( ax^2 + c = 0 \)
If \( b=0 \) & \( c=0 \): \( ax^2 = 0 \)

Question: Since “quad” implies “four,” why don’t QuadEqs involve \( x^4 \) instead of \( x^2 \)?

Answer: “Quad” comes from the Latin “quadrate” which means “squared numbers.” Also, “quadrus” means “square.” FYI: Equations with \( x^4 \) are called Quartic Equations.

Nature of QuadEqs

QuadEqs represent things that occur or change at variable rates.

QuadEq Example

Through observation and experiment, scientists devised a quadratic equation that gives the height (at any time during its flight) of an object shot or thrown straight up into the air. They named it the Position Function.

\[ h(t) = -16t^2 + vt + h \]

\( h(t) \) = height (feet) as a function of time in flight
\(-16\) = gravitational pull (feet/second per second)
\( t \) = time (seconds)
\( v \) = initial velocity (feet/second)
\( h \) = initial height above ground (feet)

Problem: A cannonball is shot straight up from the ground. Its initial velocity is 160 feet/second, but it’s slowed by the pull of gravity, stops, reverses direction, and returns to earth. How long was its flight?

Analysis: The cannonball starts on the ground, so its initial height \( h \) is 0 feet. When it lands after \( t \) seconds, its height \( h(t) \) as a function of time is also 0 feet.

If we reverse the Position Function, the problem neatly fits into a standard form QuadEq with \( a=-16 \), \( x^2=t^2 \), \( b=v \), \( x=t \), \( c=h \):

\[ \begin{align*}
ax^2 + bx + c &= 0 \\
-16t^2 + vt + h &= h(t) \\
-16t^2 + 160t + 0 &= 0 \\
-16t(t - 10) &= 0
\end{align*} \]

Factoring out -16t.

Either \( t \) makes the equation equal 0.

Solution: The ball is launched from the ground at \( t=0 \) seconds and returns to the ground when \( t=10 \) seconds.

Although it may look like the ball’s flight path, this graph shows a “time” path. The cannonball went straight up and down.

The usual solution to a QuadEq is one or both \( x \)-intercepts, where \( f(x) = 0 \). However, the graph shows all heights at any time during flight, e.g., at 5 seconds, the cannonball reached a maximum height of 400 ft.
Analyzing Coefficients: Easy as a-b-c

Knowing the effects of each coefficient (p.14) can make it easier to visualize and graph QuadEqs.

\[ y = ax^2: \text{ almost like slope} \]

‘a’ sets direction and steepness (like the slope ‘m’ in a LinEq p.43).

- Negative ‘a’ creates an inverted parabola (bowl down).
- Positive ‘a’ creates an upright parabola (bowl up).
- Larger ‘a’ creates steeper sides (narrower bowl).
- Smaller ‘a’ creates flatter sides (wider bowl).

\[ y = ax^2 + bx: \text{ bowl over} \]

‘b’ moves the parabola’s bowl up or down & left or right.
At x = 0, ‘b’ has no effect, so the y-intercept becomes the pivot point.

\[ y = ax^2 + bx + c: \text{ intercept} \]

‘c’ is the y-intercept—where the parabola crosses the y-axis.

When x = 0:
\[ y = a(0)^2 + b(0) + c \]
\[ y = c \]

**BrainAid**
Imagine a comet being intercepted by the y-axis.

\[ \frac{-b}{2a}: \text{ x-vertex} \]

-b/2a is the x-coordinate \((x_v)\) of the vertex—which is exactly halfway between the x-intercepts.
To find the y-coordinate \((y_v)\) of the vertex, substitute \((-b/2a)\) for x and solve.
# Multiplying & Factoring Expressions

Multiplying and factoring are opposite operations.

## Traditional Techniques

<table>
<thead>
<tr>
<th>Multiply Monomial • Binomial</th>
<th>Factor Binomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(x + 2)$</td>
<td>Distribute $x$ over $(x + 2)$</td>
</tr>
<tr>
<td>$x^2 + 2x$</td>
<td>Result: binomial</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiply Binomial • Binomial</th>
<th>Factor Trinomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x + 1)(x + 2)$</td>
<td>Distribute $x$ over $(x + 2)$</td>
</tr>
<tr>
<td></td>
<td>Distribute 1 over $(x + 2)$</td>
</tr>
<tr>
<td>$x^2 + 2x + x + 2$</td>
<td>Combine ‘$x$’ terms.</td>
</tr>
<tr>
<td>$x^2 + 3x + 2$</td>
<td>Result: Trinomial</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiply + and – Binomials</th>
<th>Factor Difference of 2 Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x + 2)(x - 2)$</td>
<td>Distribute $x$ over $(x - 2)$</td>
</tr>
<tr>
<td></td>
<td>Distribute 2 over $(x - 2)$</td>
</tr>
<tr>
<td>$x^2 - 2x + 2x - 4$</td>
<td>Combine $x$ terms.</td>
</tr>
<tr>
<td>$x^2 - 4$</td>
<td>Result: Difference of 2 Squares</td>
</tr>
</tbody>
</table>

## Your turn:
Multiply or factor the following expressions.

<table>
<thead>
<tr>
<th>Multiply</th>
<th>Multiply</th>
<th>Multiply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(x + 3)$</td>
<td>$(x + 2)(x + 3)$</td>
<td>$(x + 3)(x - 3)$</td>
</tr>
<tr>
<td>Factor $x^2 + 3x$</td>
<td>Factor $x^2 + 5x + 6$</td>
<td>Factor $x^2 - 9$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiply</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x(x + 3)$</td>
<td>$2x^2 + 6x$</td>
</tr>
<tr>
<td>Factor $x^2 + 5x + 6$</td>
<td>Factor $x^2 - 9$</td>
</tr>
</tbody>
</table>

**Tip:** Factor 6 into 2 • 3.
Cat Techniques
Here are some fun and memorable ways to multiply binomials and factor trinomials.

Multiply Binomial “Eyes” into Trinomial Expression

<table>
<thead>
<tr>
<th>Raise 1st Ear</th>
<th>Raise 2nd Ear</th>
<th>Make Nose &amp; Mouth</th>
<th>Drop Tongue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply first terms</td>
<td>Multiply last terms</td>
<td>Multiply inner/outer terms</td>
<td>Add inner/outer products</td>
</tr>
</tbody>
</table>

\[
(x + 1) (x + 2)
\]

Flick middle term to top.

Your turn: Multiply the cat’s binomial eyes to create its face and a trinomial expression.

Raise 1st Ear
Multiply first terms

\[
(x + 1)
\]

Raise 2nd Ear
Multiply last terms

\[
(x + 2)
\]

Make Nose & Mouth
Multiply inner/outer terms

\[
x\]

Drop Tongue
Add inner/outer products

\[
2x
\]

\[
3x
\]

Yum!
Middle terms match!

Factor Trinomial Expression into Binomial “Eyes”

<table>
<thead>
<tr>
<th>Drop 1st Ear</th>
<th>Drop 2nd Ear</th>
<th>Nose &amp; Mouth Check</th>
<th>Tongue Taste Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor first term</td>
<td>Factor last term</td>
<td>Multiply inner/outer terms</td>
<td>Add inner/outer products</td>
</tr>
</tbody>
</table>

\[
x^2 + 3x + 2
\]

\[
(x + 1) (x + 2)
\]

Yum!
Middle terms match!

Your turn: Factor the trinomial expression to create a cat’s face with binomial eyes.

\[
x^2 + 5x + 6
\]

Drop 1st Ear
Factor first term

Drop 2nd Ear
Factor last term

Nose & Mouth Check
Multiply inner/outer terms

Tongue Taste Test
Add inner/outer products
## Cat Traps & Tips

### Factoring Trap: Cat Won’t Eat!

<table>
<thead>
<tr>
<th>Cat won’t eat!</th>
<th>List Ingredients</th>
<th>Prepare Food</th>
<th>Feed Cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of inner/outer products doesn’t match middle term.</td>
<td>List all possible factors for first and last terms.</td>
<td>Combine sets of factors, cross multiply, and add.</td>
<td>Use a food combination that matches the middle term.</td>
</tr>
</tbody>
</table>

**Example:**

\[ 2x^2 + 7x + 6 \]

- **List Ingredients:**
  - List first term’s factors forwards and backwards so cover all combinations.
  - List in 1, 2, 3, 4,... order so don’t overlook factors.

- **Prepare Food:**
  - In this case, only the fourth combination adds to 7x.
  - \[ 2x \cdot x \times x \cdot 2x \]
  - \[ 1 \cdot 6 \times 2 \cdot 3 \]
  - \[ x \times 2x \times 2x \times x \]
  - \[ 6x \times 1 \times 2 \times 3 \]
  - \[ y = 12x \times 6x \]

- **Feed Cat:**
  - \[ 2x^2 + 7x + 6 \]

**Tip:** The factors of +4 must be negative to get a –8 in the middle.

### Your turn: Factor the trinomial and feed the cat.

<table>
<thead>
<tr>
<th>List Ingredients</th>
<th>Prepare Food</th>
<th>Feed Cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 3x^2 - 8x + 4 ]</td>
<td>List all possible factors for first and last terms.</td>
<td>Analyze the middle term to narrow down combinations.</td>
</tr>
</tbody>
</table>

**Analysis:** This problem has \( 4 \times 4 = 16 \) combinations!! But the large middle term -47x suggests we first test combinations that multiply our largest factors 6 and 8. Luckily, it took only two tries to get the right food!
Solving QuadEqs

To solve a QuadEq, put it in standard form and find its x-intercept/s (aka root/s).

Standard form sets the QuadEq to zero: \( ax^2 + bx + c = 0 \).
The value/s of \( x \) that make \( f(x) = 0 \) are the x-intercepts (\( x \) across, zero high/low).

If the parabola produced by a QuadEq touches the x-axis, the solutions are real numbers (p.6)
If the parabola does not touch the x-axis, the solutions are imaginary numbers (p.55).

<table>
<thead>
<tr>
<th>Two real-number solutions</th>
<th>One real-number solution</th>
<th>Imaginary-number solution/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two x-intercepts</td>
<td>Vertex = x-intercept</td>
<td>No x-intercepts</td>
</tr>
</tbody>
</table>

**Zero-Product Principle**

If a product is zero, at least one of its factors must be zero.

<table>
<thead>
<tr>
<th>If ( a \cdot b = 0 ) then</th>
<th>If ( (x)(x+1) = 0 ) then</th>
<th>If ( (x-1)(2x+1) = 0 ) then</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 0 ) and/or ( b = 0 )</td>
<td>( x = 0 ) and/or ( x + 1 = 0 )</td>
<td>( x - 1 = 0 ) and/or ( 2x + 1 = 0 )</td>
</tr>
<tr>
<td>( x = 0 ) and/or ( x = -1 )</td>
<td>( x = 0 ) and/or ( x = -\frac{1}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

**Your turn:** Apply the Zero-Product Principle to solve for the value/s of \( x \).

| \( 2x \ ( x + 3 \) = 0 \) | \( (x - 3) \ ( x + 2 \) = 0 \) | \( (2x - 1) \ ( 3x - 6 \) = 0 \) |
## Solving QuadEqs by Cat Factoring

### Factor to Solve \(x^2 + 7x + 12 = 0\)

<table>
<thead>
<tr>
<th>List Ingredients</th>
<th>Prepare Food</th>
<th>Feed Cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^2 + 7x + 12 = 0)</td>
<td>(x \cdot x)</td>
<td>((x + 3)(x + 4))</td>
</tr>
<tr>
<td>(x \cdot x)</td>
<td>(x + 12x)</td>
<td>(x = -3)</td>
</tr>
<tr>
<td>(1 \cdot 12)</td>
<td>(2 \cdot 6)</td>
<td>(x = -4)</td>
</tr>
<tr>
<td>(2 \cdot 6)</td>
<td>(3 \cdot 4)</td>
<td>(x = -4)</td>
</tr>
<tr>
<td>(3 \cdot 4)</td>
<td>(3x + 4x)</td>
<td>(x = -4)</td>
</tr>
</tbody>
</table>

Apply Zero-Product Principle

\[(x + 3)(x + 4) = 0\]

Check Solution/s

\[x + 3 = 0\]
\[-12 + 12 = 0\]
\[0 = 0 \checkmark\]
\[x = -3\]

\[x + 4 = 0\]
\[16 - 28 + 12 = 0\]
\[-12 + 12 = 0\]
\[0 = 0 \checkmark\]

\[x = -4\]

### Your turn: Factor and solve.

<table>
<thead>
<tr>
<th>List Ingredients</th>
<th>Prepare Food</th>
<th>Feed Cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^2 + 6x + 9 = 0)</td>
<td>(x \cdot x)</td>
<td>(x^2 + 7x + 12 = 0)</td>
</tr>
<tr>
<td>(x \cdot x)</td>
<td>(x + 12x)</td>
<td>((x + 3)(x + 4))</td>
</tr>
<tr>
<td>(1 \cdot 12)</td>
<td>(2 \cdot 6)</td>
<td>(x = -3)</td>
</tr>
<tr>
<td>(2 \cdot 6)</td>
<td>(3 \cdot 4)</td>
<td>(x = -4)</td>
</tr>
<tr>
<td>(3 \cdot 4)</td>
<td>(3x + 4x)</td>
<td>(x = -4)</td>
</tr>
</tbody>
</table>

Apply Zero-Product Principle

\[(x + 3)(x + 4) = 0\]

Check Solution/s

\[x^2 + 6x + 9 = 0\]
\[(-3)^2 + 7(-3) + 12 = 0\]
\[9 - 21 + 12 = 0\]
\[-12 + 12 = 0\]
\[0 = 0 \checkmark\]

\[x = -3\]

### Tip:

There is only one solution for \(x\).
Solving QuadEqs with Quadratic Formula

A QuadEq that can’t be factored is called prime.
Use the Quadratic Formula to discover its x-intercepts.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

This complicated-looking formula was derived from \( ax^2 + bx + c = 0 \) using a process called Completing the Square. It looks scary, but it’s simple to use: Substitute the values of the coefficients \( a, b, \) and \( c, \) then evaluate the expression. The result will be the x-intercept/s.

**Discriminant** [di-SKRI-mi-nunt]: \( b^2 - 4ac \)

If the discriminant (the expression inside the square root radical sign) evaluates to a:

- Perfect square (e.g., \( 0, 1, 4, 9, 16... \))—Solutions are rational real numbers (p.6).
- Positive number (e.g., \( 2, 3, 5, 6... \))—Solutions are irrational real numbers (p.6).
- Negative number (e.g., \( -1, -2, -3, -4... \))—Solutions are imaginary/complex numbers (p.55).

### When factoring won’t work...

<table>
<thead>
<tr>
<th>( x^2 + 6x + 7 = 0 )</th>
<th>( (x + 1)(x + 7) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( 7x )</td>
</tr>
<tr>
<td>( 8x )</td>
<td></td>
</tr>
<tr>
<td><strong>Yuch!</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Tip**
Always write out the coefficient values, paying special attention to + or − signs.

<table>
<thead>
<tr>
<th>( a = 1, b = 6, c = 7 )</th>
<th>( x = -6 \pm \sqrt{36 - 28} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 2 )</td>
</tr>
</tbody>
</table>

The solutions are irrational real numbers.

### Your turn: When factoring won’t work...

<table>
<thead>
<tr>
<th>( x^2 + 3x - 2 = 0 )</th>
<th>( (x - 1)(x + 2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -x )</td>
<td>( -x )</td>
</tr>
<tr>
<td>( -x )</td>
<td>( -x )</td>
</tr>
<tr>
<td><strong>Yuch!</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( a = _____, b = _____, c = _____ )</th>
<th>( x = -6 \pm \sqrt{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 2 )</td>
</tr>
</tbody>
</table>

Components of the Quadratic Formula

Imaginary/Complex Number Solutions

If the discriminant evaluates to a negative number, the QuadEq has an imaginary number solution.

**Imaginary Number i**

When first encountered, \(\sqrt{-1}\) was thought to be an impossibility, because squaring a root was always thought to produce a positive square, e.g., \(1 \cdot 1 = +1\) and \(-1 \cdot -1 = +1\).

And yet, \(\sqrt{-1} \cdot \sqrt{-1} = -1\).

So, to contrast it with the real numbers (rational and irrational), \(\sqrt{-1}\) was dubbed an “imaginary” number and represented by the italicized variable \(i\).

\[
i = \sqrt{-1}
\]

In a sense, all numbers are “imaginary” because they only represent what is real. But in fact, \(i\) does represent real phenomena that occur in nature, particularly in the area of subatomic particles.

**Complex Number**

A complex number consists of a real number and an imaginary number. Example: \(3 + i\)

<table>
<thead>
<tr>
<th>Number-Type Chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural/Whole/Integer</td>
</tr>
<tr>
<td>Fractions/Decimals/Percents</td>
</tr>
<tr>
<td>Rational</td>
</tr>
<tr>
<td>Irrational</td>
</tr>
<tr>
<td>Real</td>
</tr>
<tr>
<td>Imaginary</td>
</tr>
<tr>
<td>Complex</td>
</tr>
</tbody>
</table>

I do exist!
Word Problems

Of all areas of math, word problems (aka story problems) cause the most headaches. Why? Because they’re written in words! It’s sometimes tough to translate imprecise English words into precise math symbols. For all the anxiety they cause, I sometimes call word problems “worry” problems. But if you like to solve puzzles, this is where the fun begins!

Word Problem IDEAS

Use IDEAS to Identify/Draw/Equate/Assign/Solve word problems.

<table>
<thead>
<tr>
<th>IDEAS</th>
<th>Explanation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify</td>
<td>Identify the problem type. Nothing will aid you more in finding a solution. See Word Problem Types (p.58) for a list and references to page numbers with examples.</td>
<td>Problem: How much did Sam pay for three $2 beach balls? Type: Cost problem CPK (p.71)</td>
</tr>
<tr>
<td>Draw</td>
<td>Draw simple pictures or symbols of the items in the problem. Label values and units of measure. This will help you “see” beyond the words, which can be confusing.</td>
<td>Cost paid equals price for one ball times the quantity of balls.</td>
</tr>
<tr>
<td>Equate</td>
<td>Equate the given and unknown values into a “word” equation. Use the English-to-Math Chart (p.57) as needed. Underline sets of words that represent values.</td>
<td></td>
</tr>
<tr>
<td>Assign</td>
<td>Assign a variable to each set of underlined words in the “word” equation. Predefined equations may use specific variables.</td>
<td>C = PK</td>
</tr>
<tr>
<td>Solve</td>
<td>Solve for the unknown variable/s by plugging in given values, including units. Keep items vertically aligned. Circle the answer/s. Unit Analysis: Make sure the units of measure work out appropriately (p.60). Convert Units: As needed (p.75). Check: Plug values back into the equation/s to verify your answer/s.</td>
<td>C = $2/ball (3 balls) C =$6 Check C = PK 6 = 2(3) 6 = 6 \checkmark</td>
</tr>
</tbody>
</table>

Although you can probably solve most of the purposely-simple demonstration problems that follow without doing so, take time to complete each of the IDEAS steps, so that you’ll be prepared to set up and solve more complex problems you may encounter in the future.
English-to-Math Chart

This chart lists words used in word problems and their math equivalents.
Add more examples to the chart as you encounter them.

One of the major hurdles you’ll encounter in word problems is the tremendous number of ways that the same thing can be said with different words. And sometimes the same word can have different meanings.

For example, the word “of” can mean either multiplication or division depending on how it is used.

<table>
<thead>
<tr>
<th>ENGLISH</th>
<th>MATH</th>
<th>Sample Sentences</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ equal</td>
<td>=</td>
<td>❖ Ann is the same age as Bob.</td>
<td>A = B</td>
</tr>
<tr>
<td>➢ is</td>
<td></td>
<td>❖ Ann and Bob are equal in height.</td>
<td></td>
</tr>
<tr>
<td>➢ are</td>
<td></td>
<td>❖ Ann has as many items as Bob.</td>
<td></td>
</tr>
<tr>
<td>➢ has</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ had</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ add</td>
<td>+</td>
<td>❖ Cal has 3 more items than Deb.</td>
<td>C = D + 3</td>
</tr>
<tr>
<td>➢ sum</td>
<td></td>
<td>❖ Cal is 3 years older than Deb.</td>
<td></td>
</tr>
<tr>
<td>➢ plus</td>
<td></td>
<td>❖ Cal’s share increased by 3 over Deb’s.</td>
<td></td>
</tr>
<tr>
<td>➢ more</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ greater</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ older</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ increased by</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ subtract</td>
<td>−</td>
<td>❖ Earl has 4 items fewer than Fran.</td>
<td>E = F – 4</td>
</tr>
<tr>
<td>➢ difference</td>
<td></td>
<td>❖ Earl is 4 years younger than Fran.</td>
<td></td>
</tr>
<tr>
<td>➢ minus</td>
<td></td>
<td>❖ Earl got what was left after Fran used 4.</td>
<td></td>
</tr>
<tr>
<td>➢ less</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ fewer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ younger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ remainder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ left</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ multiply</td>
<td>•</td>
<td>❖ Gene has 5 times what Hal has.</td>
<td>G = 5H</td>
</tr>
<tr>
<td>➢ product</td>
<td></td>
<td>❖ Gene has 5 times as many as Hal.</td>
<td></td>
</tr>
<tr>
<td>➢ times</td>
<td></td>
<td>❖ Gene bought 5 items @ $H each.</td>
<td></td>
</tr>
<tr>
<td>➢ times as many as</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ @ (at)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ increased by a factor of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ divide</td>
<td>/</td>
<td>❖ Ida’s share was Jo’s share divided by 6.</td>
<td>I = J/6</td>
</tr>
<tr>
<td>➢ quotient</td>
<td></td>
<td>❖ Ida’s share equals Jo’s split 6 ways.</td>
<td></td>
</tr>
<tr>
<td>➢ split</td>
<td></td>
<td>❖ Ida equals Jo’s reduced by a factor of 6.</td>
<td></td>
</tr>
<tr>
<td>➢ per</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ reduced by a factor of</td>
<td>/</td>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ fraction of</td>
<td>•</td>
<td>❖ Gene has half of what Hal has.</td>
<td>G = ½H</td>
</tr>
<tr>
<td>➢ whole number of</td>
<td>/</td>
<td>❖ Ken has 2 of 3 items.</td>
<td>K = 2/3</td>
</tr>
</tbody>
</table>
Word Problem Types

Many word problems use predefined equations that are based on patterns discovered in nature, math, or science. Below are some of the more common equation patterns and their problem types.

<table>
<thead>
<tr>
<th><strong>Q=RK (kyu-rik) Problems</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q</strong>: Quantity</td>
</tr>
<tr>
<td><strong>R</strong>: Rate of change of Q/K</td>
</tr>
<tr>
<td><strong>K</strong>: Kwantity (made-up word)</td>
</tr>
<tr>
<td>Q = R K</td>
</tr>
<tr>
<td>Q = Q/K dissolve, K units</td>
</tr>
<tr>
<td>Q = Q K units</td>
</tr>
<tr>
<td><strong>Alternate equations</strong></td>
</tr>
<tr>
<td>R=Q/K; K=Q/R</td>
</tr>
</tbody>
</table>

**Travel Problems**

D=RT (p.64)

Distance = Rate of travel • Time

D=MV (p.67)

Distance = Mileage rate • Volume

(Miles = Miles/Gallon • Gallons)

**Cost Problems**

C=PK (p.71)

Cost = Price rate • Kwantity

(Cost = Price/Unit • Units)

**Work Problems**

W=RT (p.73)

Work = Rate of work • Time

(Work = Work/Time • Time)

**Coin Problems**

T=VC (p.72)

Total value = Value of coin • Coin quantity

(Value = Value/Coin • Coins)

**Conversion Problems**

N=CO (p.75)

New units = Conversion Rate • Old units

(New = New/Old • Old)

**Physical Problems**

W=El†

Weight = Each’s weight • Items

(Weight = Weight/Item • Items)

M=DV†

Mass = Density • Volume

(Mass = Mass/Volume • Volume)

V=FT†

Volume = Fill rate • Time

(Volume = Volume/Time • Time)

**Q=PK (kyu-pik) Problems**

**Q**: Quantity

**P**: Percent

**K**: Kwantity (made-up word)

Q has no units, so Q & K have the same units.

**Alternate equations**

P=Q/K; K=Q/P

**Interest Problems**

I=RP (p.72)

Interest = Rate of return • Principal

($ Income = Percent • $ Invested)

**Mixture Problems**

V=AT (p.74)

Volume = Amount • Total

(Volume = Volume • VolumeTotal)

**Q=K₁K₂ (kyu-kik) Problems**

**Q**: Quantity

**K₁**: Kwantity 1

**K₂**: Kwantity 2

If K₁, K₂ use same units, Q=units²

If K₁, K₂ use different units, Q=unit₁ • unit₂

**Alternate equations**

K₁=Q/K₂; K₂=Q/K₁

**Area of Rectangle**

A=WL (p.63)

(Area = Width • Length)

**Electrical Power**

E=KH†

(Energy = Kilowatts • Hours)

**Tip**

As you encounter other problem types, add them to this page, or insert an additional sheet of paper to record them.

Trap!

Textbooks use a wide variety of variables for predefined equations. Often the same variable is used to represent different items, e.g., ‘P’ can represent Percent, Price, Perimeter, Principal, etc.; ‘R’ can represent various rates, like speed, work, or percent.

Equation BrainAids

Most variables on this page were chosen and arranged to make it easier to remember the equations.

See the individual BrainAids on the referenced pages.

Other Types

**Freeform Problems**

1EqUnk/2EqUnk (p.61)

**Markup Problems**

N=O+MO (p.68)

New = Old + Markup% • Old

**Discount Problems**

N=O-DO (p.69)

New = Old – Discount% • Old

**Percent-Change Problems**

P = (N-O)/O (p.70)

Percent-change = (New – Old) / Old

† Problem types without page numbers are listed here for your use, but no examples follow.
**Word Problem Analysis**

To be solvable, a word problem must either be a 1EqUnk (p.25) or provide enough information for you to reduce more complex equations to 1EqUnks.

### Extracting Gold

Imagine a muddy stream (word problem) with a dense jumble of rocks (words) containing traces of gold (1EqUnks).

Some gold is on the surface of the rocks and easily extracted.

Other gold is embedded in the rocks and requires special extraction tools.

In each type of equation on this page, the gold 1EqUnk is shaded.

---

### 1EqUnk

\[ x + 2 = 6 \]

One Equation with One Unknown contains loose gold, which requires no special tools to extract.

\[ x + 2 = 6 \]
\[ -2 \]
\[ x = 4 \]

---

### 2EqUnk

\[ x + y = 3 \]
\[ x - y = 1 \]

Two Equations with Two Unknowns contain loose gold, which requires no special tools to extract.

\[ x + y = 3 \]
\[ x = 2 \]
\[ 2 \]
\[ 2x = 4 \]
\[ 2 \]
\[ x = 2 \]

\[ 2 + y = 3 \]
\[ 2 \]
\[ 2 \]
\[ y = 1 \]

---

### 1Eq3Unk

\[ A = B + C \]

One Equation with Three Unknowns has embedded gold that requires one of the following toolkits to extract.

**Toolkit 1**

2 values

B = 6, C = 4

\[ A = B + C \]
\[ 10 = C + 2 + C \]
\[ 10 = 2C + 2 \]
\[ -2 \]
\[ 8 = 2C \]
\[ 2 \]
\[ 4 = C \]

**Toolkit 2**

1 value

1 substitution

A = 10

\[ A = B + C \]
\[ 10 = B + 4 \]
\[ -4 \]
\[ 6 = B \]

---

### 2Eq6Unk

\[ Q_1 = R_1K_1 \]
\[ Q_2 = R_2K_2 \]

Two Equations with Six Unknowns have deeply embedded gold that requires one of the following toolkits to extract.

**Toolkit 1**

4 values

\[ R_1 = 2, K_1 = 6 \]
\[ R_2 = 3, K_2 = 4 \]

\[ Q_1 = R_1K_1 \]
\[ Q_1 = 2 \]
\[ 6 \]
\[ 12 \]

\[ Q_2 = R_2K_2 \]
\[ Q_2 = 3 \]
\[ 4 \]
\[ 12 \]

**Toolkit 2**

3 values

1 equality

\[ R_1 = 2, R_2 = 3, K_1 = 4 \]

\[ Q_1 = Q_2 \]

\[ Q_1 = Q_2 \]

\[ R_1K_1 = R_2K_2 \]

\[ 2K_1 = 3 \cdot 4 \]
\[ 2 \]
\[ 12 \]

\[ 2K_2 = 3 \cdot 4 \]
\[ 2 \]
\[ 12 \]

\[ K_1 = 6 \]

**Toolkit 3**

2 values

1 equality

1 substitution

\[ R_1 = 2, R_2 = 3 \]

\[ Q_1 = Q_2 \]

\[ K_1 = K_2 + 2 \]

\[ Q_1 = Q_2 \]

\[ R_1K_1 = R_2K_2 \]

\[ 2K_1 = 3 \cdot K_2 \]

\[ 2K_2 + 4 = 3 \cdot K_2 \]

\[ 2K_2 - 2K_2 \]

\[ 4 = K_2 \]

\[ K_1 = K_2 + 2 \]

\[ K_1 = 4 + 2 \]

\[ K_1 = 6 \]

**FYI**

1 equality + 2 substitutions or 2 equalities + 1 substitution may produce quadratic equations.
Unit Analysis

Unit Analysis can help you decide how to set up an equation to get the desired result. It ensures that your final answer will have the appropriate units before you spend time calculating.

Consider the following, almost identical problems. You probably know that division is involved, but the dilemma is: Which way to divide? Unit analysis makes it much easier to decide.

<table>
<thead>
<tr>
<th>If 50 books cost $25, how much does one book cost?</th>
<th>If 50 books cost $25, how many can you buy for $1?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Analysis: cost/book</td>
<td>Unit Analysis: books/cost</td>
</tr>
<tr>
<td>Divide: $25 cost / 50 books</td>
<td>Divide: 50 books / $25 cost</td>
</tr>
<tr>
<td>Solution: $0.50 cost / 1 book</td>
<td>Solution: 2 books / $1 cost</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>If 50 books cost $25, how much do 4 books cost?</th>
<th>If 50 books cost $25, how many can you buy for $4?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let C = Cost</td>
<td>Let B = Books</td>
</tr>
<tr>
<td>$25</td>
<td>50 books</td>
</tr>
<tr>
<td>50 books</td>
<td>$25</td>
</tr>
<tr>
<td>$25(4 books)</td>
<td>50 books($4)</td>
</tr>
<tr>
<td>50 books</td>
<td>$25</td>
</tr>
<tr>
<td>$2</td>
<td>8 books</td>
</tr>
<tr>
<td>$C</td>
<td>B books</td>
</tr>
</tbody>
</table>

Tip

In general, the item you are seeking goes on top (numerator) and the per-unit item goes on the bottom (denominator).

Proportional Ratios

In the preceding problems, dividing with the given numbers (50 books and $25) produced correct answers because both problems asked for a quantity for one; i.e., cost for one book, books for one dollar. Since the result of a division is a one in the denominator, straight division worked.

When a problem asks for more than one in the denominator, use Proportional Ratios.
Freeform Word Problems

Instead of predefined formulas, some word problems require you to build equations directly from the text in the problem. This is when the English-to-Math Chart (p.57) helps the most.

Freeform problems often involve 1EqUnk (p.25) or 2EqUnk (p.35) equations.

1EqUnk Problems

Joe bought 4 cans of nuts and gave half away as gifts. How many does he have left?

Identify: Freeform 1EqUnk problem

Draw:
- Equate: Cans left = cans bought – ½ cans bought.
- Assign: C = 4 – ½ (4)
- Solve: C = 4 – 2

Joe has 2 cans left.

Your turn: Meg picked 6 plums, gave 2 away, and picked 4 more. How many does she have now?

I
D
E
A
S

2EqUnk Problems

Tom is 3 years younger than Sue. Together they are 13 years old. How old is each?

Identify: Freeform 2EqUnk problem

Draw:
- Equate: Tom yrs = Sue yrs – 3 yrs
- Assign: T = S – 3
- Solve: T = 8 – 3

Tom is 5 years old and Sue is 8 years old.

Your turn: Bob has 2 more pens than Jan. Together they have 10. How many pens does each have?

I
D
E
A
S
# Geometric Word Problems

**Perimeter Problems**

Perimeter [pur-IH-meh-tur] is a measure of the distance around an object. *Peri* is Greek for “around.” *Meter* is Greek for “measure.”

## Rectangle

The perimeter of a rectangle is twice its length plus twice its width.

\[ P_R = 2L + 2W \]

<table>
<thead>
<tr>
<th>L</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (long side)</td>
<td>Width (short side)</td>
</tr>
</tbody>
</table>

**Alternate Equation:** Factoring out the 2 yields: \( P_R = 2(L + W) \)

## Square

The perimeter of a square is four times the length of one side.

\[ P_S = 4S \]

<table>
<thead>
<tr>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of one side</td>
</tr>
</tbody>
</table>

## Circle

The perimeter, aka circumference [sur-CUM-frenss], of a circle is its Diameter times pi [pii].

\[ C = \pi d \]

| \( \pi \) = Pi \| d = Diameter (a line through the center) |
|---|---|
| \( \approx 3.14 \) or \( \approx 22/7 \) | |

*The word pi is Greek for *periphery* and came from measuring circles.*

## How much fencing is needed to enclose a 100 ft by 50 ft field? (ft = foot or feet)

**Identify:** Rectangle Perimeter problem

**Draw:**

<table>
<thead>
<tr>
<th>( P_R )</th>
<th>( L )</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Assign:</td>
<td>( P_R = 2L + 2W )</td>
<td></td>
</tr>
<tr>
<td>Solve:</td>
<td>( P_R = 2(100) + 2(50) )</td>
<td>( 300 \text{ ft} )</td>
</tr>
</tbody>
</table>

**Your turn:** What is the distance around a village square that’s 30m on each side? (m = meters)

I

D

E

A

S
Area Problems

Area [AIR-ee-uh] is a measure of the space on the surface of an object. Area is Latin for “level ground” or “open space.”

**Rectangle**
The area of a rectangle is its width times its length.
\[ A_R = WL \]
\[ A_R = \text{Area of rectangle} \]
\[ W = \text{Width (short side)} \]
\[ L = \text{Length (long side)} \]

**Square**
The area of a square is the length of one side squared.
\[ A_S = S^2 \]
\[ A_S = \text{Area of square} \]
\[ S = \text{Length of one side} \]

**Circle**
The area of a circle is pi times its radius squared.
\[ A_C = \pi r^2 \]
\[ A_C = \text{Area of circle} \]
\[ r = \text{radius (a line from center to edge =} \frac{1}{2} \text{ diameter)} \]
\[ \pi = \text{pi} \approx 3.14 \text{ or } \approx \frac{22}{7} \]

---

How many square feet is a circular lawn whose radius is 10 ft? (ft = foot or feet, \( \text{ft}^2 = \text{square ft} \))

Identify: Circle Area problem

Draw:

\[ \text{Equate: Area}_{\text{circle}} = \frac{\pi \cdot \text{radius}^2}{\pi r^2} \]
\[ \text{Assign: } A_C = \frac{\pi \cdot 10^2}{10^2} \]
\[ \text{Solve: } A_C = 3.14(10)^2 \]
\[ A_C = 314 \text{ ft}^2 \]

How many bags of seed are needed for this lawn if one bag covers 157 \( \text{ft}^2 \)?

Identify: Freeform Division problem

Draw:

\[ \text{Equate: Bags = Area}_{\text{Lawn}} \div \text{Coverage}_{\text{Seed}} \]
\[ \text{Assign: } B = A \div C \]
\[ \text{Solve: } B = \frac{314}{157} \]
\[ B = 2 \text{ bags} \]

Your turn: How many square yards is a tarp that measures 50 yd x 30 yd? (yd = yard/s, \( \text{yd}^2 = \text{square yd} \))

I

D

E

A

S

Travel Word Problems

Distance/Rate/Time: DRT

The Distance traveled equals the Rate of travel times the Time traveled.

\[ D = RT \]

- **D** = Distance traveled
- **R** = Rate of travel (average speed)
- **T** = Time traveled

**Travel Rate:** \[ R = \frac{D}{T} \]

**Travel Time:** \[ T = \frac{D}{R} \]

**What distance is traveled by a biker averaging 10 mph for 2 hours?** (mph = miles per hour)

- **Identify:** Travel Distance problem
- **Draw:**
  - Equate: \( D_{\text{mi}} = \text{Rate}_{\text{mph}} \times \text{Time}_{\text{hr}} \) (mi=mile/s, hr=hour/s)
  - Assign: \( D = RT \)
  - Solve: \( D = 10 \text{ miles/hr} \times 2 \text{ hours} = 20 \text{ miles} \)

- **A biker who rides 60 miles in 4 hours pedals how fast on average?**
  - **Identify:** Travel Rate problem
  - **Draw:**
    - Equate: \( \text{Rate}_{\text{mph}} = \frac{D_{\text{mi}}}{\text{Time}_{\text{hr}}} \)
    - Assign: \( R = \frac{D}{T} \)
    - Solve: \( R = \frac{60 \text{ miles}}{4 \text{ hours}} = 15 \text{ mph} \)

- **How long does a biker take to ride 12 miles at 3 mph?**
  - **Identify:** Travel Time problem
  - **Draw:**
    - Equate: \( \text{Time}_{\text{hr}} = \frac{D_{\text{mi}}}{\text{Rate}_{\text{mph}}} \)
    - Assign: \( T = \frac{D}{R} \)
    - Solve: \( T = \frac{12 \text{ miles}}{3 \text{ miles/hour}} = 4 \text{ hours} \)

**Your turn:** How far does a biker ride when averaging 15 mph for 5 hours?

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**S**
Double DRT: Round Trip Average Rate
The average Rate of travel for a round trip (or several shorter trips) is the total Distance divided by the total Time. \( R_{\text{avg}} = \frac{D_{\text{total}}}{T_{\text{total}}} \)

What is the average rate of a car that travels 60 miles outbound @ 60 mph, then returns 60 miles inbound @ 30 mph? (\( \text{mi} = \text{mile/s}; \text{mph} = \text{miles per hour, hr = hour/s} \))

**INCORRECT**
Identify: Average = Sum of Items / Total Items
Draw:
Equate: \( R_{\text{avg}} = \frac{R_{\text{out}} + R_{\text{in}}}{2} \)
Assign: \( R_A = \frac{R_O + R_I}{2} \)
Solve: \( R_A = \frac{(60\text{mph} + 30\text{mph})}{2} = 90 \text{mph} / 2 = 45 \text{mph} \)

**CORRECT**
Identify: Travel Time problem (two trips)
Draw:
Equate: \( T_O = \frac{D_O}{R_O} \quad T_I = \frac{D_I}{R_I} \)
Assign: \( T_O = \frac{60\text{ mi}}{60\text{ mph}} = 1 \text{ hr} \quad T_I = \frac{60\text{ mi}}{30\text{ mph}} = 2 \text{ hr} \)

Identify: Round Trip Average Rate
Draw:
Equate: \( R_{\text{avg}} = \frac{D_{\text{total}}}{T_{\text{total}}} \quad R_{\text{avg}} = \frac{D_{\text{total}}}{T_{\text{total}}} \)
Assign: \( R_A = \frac{D_O + D_I}{T_O + T_I} \)
Solve: \( R_A = \frac{60\text{ mi} + 60\text{ mi}}{1 \text{ hr} + 2 \text{ hr}} = \frac{120 \text{ mi}}{3 \text{ hr}} = 40 \text{ mph} \)

**Trap!**
45 mph, the midpoint between 60 mph and 30 mph, would be correct if the car traveled the same amount of time in both directions. But the car necessarily took longer to cover the inbound 60 miles at the slower 30 mph, which pulled the average down to 40 mph.

**Your turn:** What is the average rate of a car than travels 30 miles outbound @ 30 mph, then returns 30 miles inbound @ 10 mph?

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**Double DRT: Catch Up**

Bus#1 leaves the depot and averages 50 mph. Bus#2 leaves 1 hour later averaging 75 mph. How long will it take Bus#2 to catch up to Bus#1?

At what distance from the depot?

Identify: Travel Catch Up problem (Distance equal)

Draw:

Equate: Distance_1 = Distance_2  
Time_1 = Time_2 + 1 hr  
Rate_1 = 50mph  
Rate_2 = 75mph

Assign:  
R_1T_1 = R_2T_2  
R_1(T_2+1) = R_2T_2

Solve:  
50mph(T_2+1hr) = 75mph(T_2)  
D_2 = R_2T_2

50T_2mi + 50mi = 75T_2mi  
D_2 = 75mph • 2hr

-50T_2mi = -50T_2mi  
D_2 = (150 mi)

50 mi = 25T_2 mi  
25mph 25mph

2 hr = T_2

**Solution**  
Bus2 catches up to Bus1 in 2 hours 150 miles from the depot.

---

**Your turn:** Ann leaves school and walks 2 mph. Bob leaves 1 hour later and walks 4 mph. How long will it take him to catch up with Ann and at what distance from school?

I

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Double DRT: Meet in Between

Eve and Jim are 10 miles apart and start walking towards each other. Eve walks 2 mph. Jim walks 3 mph. How long does it take, and how far has each walked when they meet?

Identify: Travel Meet in Between (Time equal)

Draw:

Equate: \( \text{Time}_{E} = \text{Time}_{J} = T \)

Assign: \( D_{E} = R_{E}T \)

Add equations to save steps

\[ D_{E} + D_{J} = (R_{E} + R_{J})(T) \]

Solve: \( 10 \text{ mi} = (2 + 3) \text{ mph}(T) \)

\[ \frac{10 \text{ mi}}{5 \text{ mph}(T)} = \frac{2 \text{ hr}}{T} \]

They meet in 2 hours. Ann walked 4 miles; Bob walked 6 miles.

Double DRT Variations

Many variations of distance, rate, and time and the relationships between them are possible, e.g., travelers may leave at same or different times, total distance may be provided but not individual distances, rates may be given in terms of each other as in “twice as fast.” The variations seem endless but are always based on \( D = RT \).

Tip

See Word Problem Analysis (p.58) for the minimum elements needed to solve 2Eq6Unk problems like Double DRTs.

Mileage: DMV

Distance traveled equals Mileage rate times fuel Volume.

\[ D = MV \]

\( D = \) Distance in miles

\( M = \) Mileage in miles per gallon (mpg)

\( V = \) Volume of fuel in gallons (gal)

Alternate Equations

\[ M = D/V \]

\[ V = D/M \]

A car travels 500 miles on a 20-gallon tank of gas. What is its mpg?

Identify: Travel Mileage

Draw:

Equate: \( \text{mpg} = \frac{\text{Distance}_{\text{mi}}}{\text{Volume}_{\text{gal}}} \)

Assign: \( M = D/V \)

Solve: \( M = \frac{500 \text{ mi}}{20 \text{ gal}} = 25 \text{ mpg} \)
Financial Word Problems

Besides financial items, these equations will work for almost any type of percent increase, decrease, or change problems. Tip: Review percents, fractions, and decimals in Max Learning’s Fraction Fun.

**Price Markup on Cost: NO+MO**

Merchants mark up (raise) the price of a product so they can make a profit on each sale.

The New price equals the Old price plus the Markup% times the Old price.

\[ N = O + MO \]

- **N** = New price (aka Retail or List price)
- **O** = Old price (aka Wholesale or Original price)
- **M** = Markup Percent

Alternate Equation: Factoring out the **O** yields: \[ N = O(1 + M) \].

Explanation: \((1 + M)\) is the multiplier that yields the New price, e.g., if Markup = 20%, the New price is 120% of the Old.

The math:

\[(1 + 20\%) = (100\% + 20\%) = 120\%\]

**What is the price of a $10 coat after a 20% markup?**

Identify: Price Markup problem

Draw:

\[ \text{Equate: New price} = \text{Old price} + \text{Markup}\% \cdot \text{Old price} \]

Assign: \(N = O + MO\)

Solve: \(N = 10 + 20\%(10) = 10 + 2 = 12\)

**Your turn:** What is the price of a $20 coat after a 50% markup?

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**Tax “Markup”: NO+TO**

Adding sales tax to an item is like marking it up by the tax percentage.

\[ N = O + TO \]

- **T** = Tax percentage (replaces **M**)

**What is the price of a $12 coat with 5% sales tax?**

Identify: Tax Markup problem

Draw:

\[ \text{Equate: New price} = \text{Old price} + \text{Tax}\% \cdot \text{Old price} \]

Assign: \(N = O + TO\)

Solve: \(N = 12 + 5\%(12) = 12 + 0.60 = 12.60\)

**BrainAid**

Imagine a positive (+) merchant named NO+MO [noh-moh] who loves to Markup prices.

**Tip**

To distinguish between the New markup price and the New price after tax, use different subscripts for **N**, e.g.,

\(N_M = \text{New markup price}\)

\(N_T = \text{New price after tax}\)
**Price Discount: NO–DO**

Merchants discount (lower) the price of a product to increase the number of items sold.

The New price equals the Old price minus the Discount\% times the Old price.

\[ N = O - D \]

- **N** = New price (aka Discounted or Sale price)
- **O** = Old price (aka Retail, List, or Original price)
- **D** = Discount Percent

**Alternate Equation:** Factoring out the O yields: \( N = O(1 - D) \).

**Explanation:** \((1 - D)\) is the multiplier that yields the New price, e.g., if Discount = 30\%, the New price is 70\% of the Old.

**The math:**

\[
(1 - 30\%) = (100\% - 30\%) = 70\%
\]

**What is the price of a $10 T-shirt after a 25\% discount?**

**Identify:** Price Discount problem

**Draw:**

**Equate:** New price = Old price – Discount\% • Old price

**Assign:** \( N = O - D \)

**Solve:** \( N = $10 - 25\%(10) = $10 - $2.50 = $7.50 \)

**Your turn:** What is the price of a $20 T-shirt after a 50\% discount?

**Your turn:** What is the price of the discounted T-shirt (from above) with 10\% sales tax?

**BrainAid**

Imagine a negative (–) merchant named NO-DO [noh-doh] who has to Discount prices.

**Shortcut Solution**

If 25\% is deducted, 75\% remains.

75\%(10) = \$7.50
Percent-Change: PN-O/O

Merchants sometimes need to calculate the percent change between two prices to determine the markup or discount percent.

The Percent change equals the difference between the New price and the Old price divided by the Old price.

\[ P = \frac{N - O}{O} \]

- \( P \) = Percent change
- \( N \) = New price (aka Current price)
- \( O \) = Old price (aka Original or Base price)

What is the percent markup on a $10 dress that now sells for $15?

**Identify:** Percent Change problem  
**Draw:**  
**Equate:** Percent-change = (New price – Old price) / Old price  
**Assign:** \( P = \frac{N - O}{O} \)  
**Solve:** \( P = \frac{($15 - $10)}{$10} = \frac{5}{10} = 50\% \)

What is the percent discount on a $15 dress that’s on sale for $10?

**Identify:** Percent Change problem  
**Draw:**  
**Equate:** Percent-change = (New price – Old price) / Old price  
**Assign:** \( P = \frac{N - O}{O} \)  
**Solve:** \( P = \frac{($10 - $15)}{$10} = \frac{-5}{10} = -50\% \)

**The minus indicates a decrease. Because a discount is also a decrease, we’d say the percent discount is 33%, not -33%.”**

Your turn: What is the percent discount on a $20 dress that’s on sale for $15?

I  
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BrainAid

Imagine PiNOcchiO, the puppet who became a boy, has the nickname PNOO [pi-noh]. He calculates the percent change in his nose size when he tells a lie.

\[ \text{Pi} = \frac{N - O}{O} \]

Percent Paradox

A larger Old price results in a smaller percent change.  
Although the difference between the New and Old prices was $5 for both markup and discount, the percent changes were not the same, because they were based on different Old prices, first $10 (50% change) then $15 (-33% change).
Cost: CPK

Merchants must often calculate the cost of selling or buying a quantity of identical items.

The Cost equals the Price per item times the Kwantity of items.

\[ C = PK \]

\[ C = \text{Cost of all items} \]

\[ P = \text{Price (aka Cost) for one item} \]

\[ K = \text{Kwantity (made-up word) of items purchased} \]

What is the total cost of 5 hammers sold for $10 each?

Identify: Cost problem

Draw:

\[ \text{Equate: Cost of hammers} = \text{price per hammer} \cdot \text{kwantity of hammers} \]

Assign: \[ C = PK \]

Solve: \[ C = \$10/\text{hammer} (5 \text{ hammers}) = (\$50) \]

If a box contained twenty $10 hammers, how much would 3 boxes cost?

Identify: Cost problem (1 of 2)

Draw:

\[ \text{Equate: Cost of box} = \text{price per hammer} \cdot \text{kwantity of hammers per box} \]

Assign: \[ C = PK \]

Solve: \[ C = \$10/\text{hammer} (20 \text{ hammers/box}) = $200/\text{box} \]

Identify: Cost problem (2 of 2)

Draw:

\[ \text{Equate: Cost of 3 boxes} = \text{price per box} \cdot \text{kwantity of boxes} \]

Assign: \[ C = PK \]

Solve: \[ C = $200/\text{box} (3 \text{ boxes}) = (\$600) \]

Your turn: What is the total cost of 4 toasters sold for $15 each?

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Also see Unit Analysis and Proportional Ratios (p.60) for alternative approaches to setting up and solving Cost and other problems.
### Interest Earned: IRP
Interest earned on an investment equals the Rate (percent) of annual interest times the Principal invested.

\[ I = RP \]

- \( I \) = Interest earned ($)  
- \( R \) = Rate of annual interest (percent)  
- \( P \) = Principal invested ($)  

**How much interest does Ron earn in one year on a $1000 investment at 6%?**

*Identify:* Interest Earned problem

*Draw:*  

*Equate:* Interest earned = interest rate \( \times \) principal  

*Assign:*  

\[ I = 6\% \times 1000 \]

*Solve:*  

\[ I = 6\% \times 1000 = \frac{6}{100} \times 1000 = \$60 \]

**I = RPT**  

- \( T \) = Time period (in years or fraction of a year)

\( I=RP \) is derived from \( I=RPT \) where \( T=1 \). But any period can be used, e.g., \( T=2 \) equals 2 years. \( T=1/12 \) equals one month.

**BrainAid:** Mr. IRP exclaims: “I reap interest RePeaTedly” over several periods.

\( I = RPT \) computes simple interest, which is calculated only on the originally invested Principal each period.

FYI: For compound interest, add the interest earned each period to the Principal, then compute the next period’s interest on the new higher total. Compounding is a good thing for the investor since it increases the total interest earned.

### Coins: TVC
The Total value of a group of the same-type coin is the Value of one coin times the number of those Coins.

\[ T = VC \]

- \( T \) = Total value  
- \( V \) = Value of one coin of that type  
- \( C \) = Coins of that type

**What is the total value of Ned’s 30 nickels?**

\[ T = .05/\text{nickel} \times 30 \text{ nickels} = \$1.50 \]

Coin problems usually combine the total values of several coin equations.  
- The variable ‘\( C \)’ changes for each coin type: \( P=\text{Pennies}, \quad N=\text{Nickels}, \quad D=\text{Dimes}, \quad Q=\text{Quarters} \).
- Total \( \text{All coins} = .01P + .05N + .10D + .25Q \)

**Peg has a total of 7 dimes and quarters worth $1. How many of each does she have?**

*Identify:* Coin problem

*Draw:*  

*Equate:*  

\[ \text{Dimes} + \text{Quarters} = 7 \]

*Assign:*  

\[  \text{Total value} = .10 \times \text{Dimes} + .25 \times \text{Quarters} \]

*Solve:*  

\[ \begin{align*}  
D + Q & = 7  
D & = 7 - Q  
D & = 7 - 2 - Q  
D & = 5 - Q \end{align*} \]

*Check:*  

\[ \begin{align*}  
.30 & = \frac{.15Q}{2}  
.15 & = \frac{.15Q}{2}  
Q & = 2 \end{align*} \]
## Work Word Problems

### Work/Rate/Time: WRT

The Work completed equals the Rate of work times the Time worked.

\[ W = RT \]

- **W** = Work completed (aka job, task)
- **R** = Rate of work
- **T** = Time worked

**Alternate Equations:**
- \( R = \frac{W}{T} \)
- \( T = \frac{W}{R} \)

**How many tasks can Rob complete if he performs 1 task in 2 hours and works for 10 hours?**

\[ W = \frac{1 \text{ task}}{2 \text{ hours}} \cdot 10 \text{ hours} = 5 \text{ tasks} \]

Work problems usually combine the work rates of more than one worker.

**Cal can paint 1 room in 2 hours. Zoe can paint 1 room in 3 hours. How long does it take them to paint 1 room together?**

**Identify:** Work problem

**Draw:**

**Equate:**

\[ W_{\text{Both}} = (R_{\text{Cal}} + R_{\text{Zoe}}) T \]

**Assign:**

\[ W = (R_C + R_Z) T \]

**Solve:**

\[
\begin{align*}
6[1\text{rm}] &= (1\text{rm}/2\text{hr} + 1\text{rm}/3\text{hr}) T \\
6\text{rm} &= (3\text{rm/hr} + 2\text{rm/hr}) T \\
6\text{rm} &= \frac{5\text{rm/hr}}{5\text{rm/hr}} T \\
1 \frac{1}{5} \text{hr} &= T
\end{align*}
\]

**Your turn:** Ona can assemble one bike in 1 hour. Mac can assemble one bike in 2 hours. How long does it take for them to assemble one bike together?

<table>
<thead>
<tr>
<th>I</th>
<th>D</th>
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</thead>
<tbody>
<tr>
<td>E</td>
<td>A</td>
</tr>
<tr>
<td>S</td>
<td></td>
</tr>
</tbody>
</table>
Mixture Word Problems

Volume/Amount/Total: VAT
The Volume of one component is an Amount (percent) of the Total mixture.

\[ V = AT \]

\( V \) = Volume of one component
\( A \) = Amount (percent)
\( T \) = Total volume of mixture

Alternate Equations: \( A = V/T \), \( T = V/A \)

How many ounces (oz) of water are in a 100 oz beaker that’s 25% water?

\( V = 25\% \times (100 \text{ oz}) = 25 \text{ oz} \)

Typical mixture problems involve changes to the volume of components.

Ethanol is 40% of a 10-pint mixture. Tim adds 2 more pints of ethanol. What is its new percent?

Identify: Mixture problem

Draw:

Equate: \( \frac{\text{Volume of ethanol}}{\text{Total volume}} = \frac{\text{Amount } \% \text{ ethanol}}{\text{Total volume}} \)

\( \frac{V_E}{V_{E2}} = \frac{A_{E}}{T_{E2}} \)

Assign: \( V_E = A_E T \)

Solve: \( V_E = 40\% \times (10 \text{ pt}) = 4 \text{ pt} \rightarrow (4 + 2 \text{ pt}) = A_{E2} \times (10 + 2 \text{ pt}) \)

\( \frac{6 \text{ pt}}{12 \text{ pt}} = A_{E2} \times \frac{12 \text{ pt}}{12 \text{ pt}} \)

\( 50\% = A_{E2} \)

Your turn: Methyl is 10% of a 100-gallon mixture. Kai adds 20 more gallons of methyl. What is its new percent?

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BrainAid
Imagine stirring a mixture in a large VAT.
Conversion Word Problems

A word problem may require you to convert one unit of measure into another.

**Conversions: NCO**

The number of New units equals the Conversion rate times the number of Old units.

\[ N = CO \]

- \( N \): New units
- \( C \): Conversion rate
- \( O \): Old units

Alternate equations: \( C = N/O \); \( O = N/C \)

The variables for \( N \) and \( O \) will change depending on the units being converted.

**Rae walked for 1.5 hours. How many minutes did she walk?**

Identify: Conversion problem

Draw:

Equate: Minutes walked = 60 minutes per hour \( \times \) hours walked

Assign: \( M = \frac{60 \text{ min}}{\text{hr}} \times H \) (min = minute/s; hr = hour/s)

Solve: \( M = \frac{60 \text{ min}}{\text{hr}} \times 1.5 \text{ hr} = 90 \text{ min} \)

**Nat bought a 96-inch piece of wood. How many feet is it?**

Identify: Conversion problem

Draw:

Equate: Feet of wood = 1 foot per 12 inches \( \times \) inches of wood

Assign: \( F = \frac{1 \text{ ft}}{12 \text{ in}} \times I \) (ft = foot/feet; in = inch/\text{s})

Solve: \( F = \frac{1 \text{ ft}}{12 \text{ in}} \times 96 \text{ in} = 8 \text{ ft} \)

**Your turn:** Eve has a 10-foot tree in her yard. How many inches tall is it?
Conversion by Replacement/Ratio

Most conversions are usually part of a more complicated word problem and don’t always merit the full IDEAS treatment. Below are two alternate conversion methods.

### Conversion by Replacement
Replace the old unit with its equivalent in the new unit and multiply.

**3 hours = ? minutes**
Process: Replace “hours” with “60 minutes” and multiply.
Solution: 3 hours = 3(60 minutes) = 180 minutes

**24 inches = ? feet**
Process: Replace “inches” with “1/12 foot” and multiply.
Solution: 24 inches = 24(1/12 foot) = 2 feet

**Why Replacement Works:**
It’s based on N=CO being reversed to OC=N.

Old units • Conversion rate = New units
3 hours • 60 minutes/hour = 180 minutes
24 inches • 1/12 foot/inch = 2 feet

### Conversion by Ratio
Set the New/Old ratio to the Conversion-rate ratio. Solve for the New unit. Tip: Use Shoot-the-Chute (p.33).

**How many seconds (S) are in 10 minutes?**

\[
\frac{S}{10\text{ min}} = \frac{60\text{ sec}}{1\text{ min}}
\]
\[
S = \frac{(10\text{ min})(60\text{ sec})}{1\text{ min}}
\]
\[
S = 600\text{ sec}
\]

**Why Ratios Work:**
They’re based on N=CO being altered to N/O = C.

**Inverse Ratios**
In problems that place the unknown variable in the denominator of the ratio, make sure the units in the conversion-rate ratio match top to bottom. See Unit Analysis (p.60).

---

**Your turn:**
1. 120 min = ? hr
   Tip: 1 min = 1/60 hr

   **Your turn:**
   2. 2 yds = ? ft

   **Your turn:**
   3. How many feet (F) are in 5 yards?
Conversion Ladders
Ladders make multistep conversions easy!

U.S. Liquid Ladder

Old Units

New Units

Equivalents
Each ladder rung links equivalent units (e.g., 4 qt = 1 Gallon) and yields a Conversion rate (e.g., C = 4 qt/Gallon).

How many teaspoons in an ounce?
Procedure: Start at 3 tsp. Climb/multiply to 2 TBS.
Solution: 3 • 2 = 6
Why it works: 3 tsp/TBS • 2 TBS/oz = 6 tsp/oz

Your turn: How many ounces in a quart?

U.S. Linear Ladder

How many feet in a mile?
3 • 1760 = 5280
Your turn: How many inches in a yard?

Time Ladder

How many seconds in a day?
60 • 60 • 24 = 86400
Your turn: How many minutes in a day?

Tip
Larger dictionaries often contain conversion charts that compare U.S. to Metric units of measure.
Look under “measure” or in the appendix.

Trap!
Some conversion charts list conversion equations like:
miles × 1.6 = kilometers
This does not mean that 1.6 miles = 1 kilometer.
It means OC = N where O = Old units = miles C = Conversion rate = 1.6 km/mi N = New units = kilometers
In fact, 1 mile = 1.6 kilometers.
Answer Key

One Equation, One Unknown

Page 25: 1EqUnk Added Term
Top Row: x = 1, x = 3; Bottom Row: x = 4, x = 9

Page 26: 1EqUnk Subtracted Term
Top Row: x = 7, x = 9; Bottom Row: x = 14, x = 21

Page 27: 1EqUnk Multiplied Variable
Top Row: x = 3, x = 2; Bottom Row: x = 5, x = 4

Page 28: 1EqUnk Divided Variable
Top Row: x = 12, x = 3; Bottom Row: x = 8, x = 15

Page 29: Multiple Operations: Clear As Mud
x = 5; x = 3

Page 30: Multiple Terms: Family Reunion
Top Row: 7x – 5, x^2 – 2x + 9; Bottom Row: -3x^2 + 5x + 1, -4x^2 – x + 3

Page 31: Separated Terms: Take Sides / Distributed Terms: Fair to All
Separated: 2x = 8. Distributed: -5x + 10 = 25, 5x – 10 = 25, x – 2 = 25

Page 32: Simplifying Coefficients: Clear Denominators / Reduce Coefficients
Clear: 5x + 5 = 2, x + 3 = 4. Reduce: 3x + 1 = 4, x + 2 = -3

Page 33: Clearing Equated Fractions: Shoot-the-Chute
x = 5/3; x = 9/14; x = 8/15

Page 34: Combining Fractions: Spotlighting
Left column: 7x/10 = 3, x/6 = 8. Right column: 5x/6 = 7, x/8 = 9

Two Equations, Two Unknowns

Page 38: Eliminate to Solve
Add: (3, 1). Subtract: (1, 2)

Page 39: Multiply Then Eliminate
Left: 10x=8 or 5y=3. Center: 3x=-13 or 3y=14. Right: 12x=25 or 12y=7

Page 40: 2EqUnk Substitution: Masquerade
(2, 4)

Linear Equations

Page 42: Slope-Intercept Form
y = 2x + 3, m = 2, b = 3

Page 44: Calculating Slope: Δy / Δx
m = 2, m = -2, m = 2

Page 46: Plotting LinEqs
See plots on this page.
Quadratic Equations
Page 49: QuadEq Traditional Techniques
Left column: $x^2 + 3x, x(x + 3), 2x^2 + 6x, 2x(x + 3)$
Center column: $x^2 + 5x + 6, (x + 2)(x + 3)$
Right column: $x^2 - 9, (x + 3)(x - 3)$

Page 50: QuadEq Cat Techniques
$x^2 + 5x + 6$  
\[
\begin{array}{c}
(\text{Left}) \\
(\text{Right})
\end{array}
\]

Page 51: QuadEq Cat Traps & Tips

<table>
<thead>
<tr>
<th>List Ingredients</th>
<th>Prepare Food</th>
<th>Feed Cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3x^2}{x} - 8x + 4$</td>
<td>$x \cdot 3x$</td>
<td>$3x^2 - 8x + 4$</td>
</tr>
<tr>
<td>$\frac{3x \cdot x}{3x \cdot x}$</td>
<td>$-1 \cdot -4$</td>
<td>$(3x - 2)(x - 2)$</td>
</tr>
<tr>
<td>$x \cdot 3x - 1 \cdot -4$</td>
<td>$-3x + -4x$</td>
<td>$-2x + -6x$</td>
</tr>
<tr>
<td>$3x \cdot x$</td>
<td>$-x + -12x$</td>
<td>$-8x$</td>
</tr>
</tbody>
</table>

Page 52: Zero-Product Principle
Left: $x = 0$ and/or $x = -3$. Center: $x = 3$ and/or $x = -2$. Right: $x = \frac{1}{2}$ and/or $x = 2$.

Page 53: Solving QuadEqs by Cat Factoring

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<td>$\frac{x^2}{x} + 6x + 9$</td>
<td>$x \cdot 9$</td>
<td>$x^2 + 6x + 9$</td>
</tr>
<tr>
<td>$\frac{1 \cdot 9}{3 \cdot 3}$</td>
<td>$3x + 3x$</td>
<td>$(x + 3) (x + 3)$</td>
</tr>
<tr>
<td>$x \cdot x$</td>
<td>$x + 9x$</td>
<td>$3x + 3x$</td>
</tr>
<tr>
<td>$x + 9x$</td>
<td>$3x + 3x$</td>
<td>$6x$</td>
</tr>
</tbody>
</table>

Apply Zero-Product Principle
$(x + 3)(x + 3) = 0$
$x + 3 = 0$
$x = -3$

Check Solution/s
$x^2 + 6x + 9 = 0$
$(-3)^2 + 6(-3) + 9 = 0$
$9 - 18 + 9 = 0$
$-9 + 9 = 0$
$0 = 0 \checkmark$

Page 54: Solving QuadEqs with Quadratic Formula
\[x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)} = \frac{-3 \pm \sqrt{17}}{2}\]
**Word Problems**

**Page 61: Freeform Word Problems**

1EqUnk: Meg plums = 6 – 2 + 4 = 8.


**Page 62: Geometric Word Problems**

Perimeter (p.62): Perimeter\(\text{square} = 4 \cdot \text{Side} = 4(30) = 120 \text{ m}\)

Area (p.63): Area\(\text{rect} = \text{Length} \cdot \text{Width} = 50 \cdot 30 = 150 \text{ yd}^2\)

**Page 64: Travel Problems**

DRT (p.64): Distance = 15 miles/hour \(\cdot\) 5 hours = 75 miles

Double DRT: Round Trip Average Rate (p.65): The average rate for the round trip was 15 mph.

\[
T_{\text{out}} = \frac{D_{\text{out}}}{R_{\text{out}}} = \frac{30 \text{ mi}}{30 \text{ mph}} = 1 \text{ hr};
T_{\text{in}} = \frac{D_{\text{in}}}{R_{\text{in}}} = \frac{30 \text{ mi}}{10 \text{ mph}} = 3 \text{ hr}
\]

\[
R_{\text{avg}} = \left(\frac{D_{\text{out}} + D_{\text{in}}}{T_{\text{out}} + T_{\text{in}}}ight) = \frac{(30 \text{ mi} + 30 \text{ mi})}{(1 \text{ hr} + 3 \text{ hr})} = 60 \text{ mi} / 4 \text{ hr} = 15 \text{ mph}
\]

Double DRT: Catch up (p.66): Bob catches up to Ann in 1 hour 4 miles from school.

\[
D_{\text{Ann}} = D_{\text{Bob}}; T_{\text{A}} = T_{\text{B}} + 1; R_{\text{A}}(T_{\text{B}} + 1) = R_{\text{B}} T_{\text{B}}; 2\text{mph}(T_{\text{B}} + 1) = 4\text{mph}(T_{\text{B}}); T_{\text{B}} = 1\text{ hr}; D_{\text{B}} = 4\text{mph} \cdot 1\text{ hr} = 4\text{mi}
\]

**Page 68: Financial Word Problems**

Price Markup on Cost (p.68): \(N = $20 + 50\%($20) = $20 + $10 = $30\)

Price Discount (p.69): \(N = $20 – 50\%($20) = $20 – $10 = $10\); \(N_{\text{tax}} = $10 + 10\%($10) = $10 + $1 = $11\)

Percent-Change (p.70): \(P = (\frac{15 – 20}{20}) / 20 = -5/20 = -25/100 = -25\%\)

Cost (p.71): \(C = $15/\text{toaster} \cdot 4 \text{ toasters} = $60\)

**Page 73: Work Word Problems**

WRT: \(W_{\text{Both}} = (R_{\text{Ona}} + R_{\text{Mac}})T_{\text{Both}}; 1\text{bk} = (1\text{bk/hr} + 1\text{bk/2hr})T; T = 2/3 \text{ hr} (\text{bk}=\text{bike})\)

**Page 74: Mixture Word Problems**

VAT: \(V_{\text{M}} = 10\%\%(100\text{gal}) = 10\text{gal}. \ (10+20\text{gal}) = A_{\text{M20}}(100+20\text{gal}). \ A_{\text{M20}} = 25\%\)

**Page 75: Conversion Word Problems**

NCO (p.75): Inches = 12in/ft \(\cdot\) 10ft = 120 inches

By Replacement (76): 120min(1/60hr) = 2hr. 2yd(3ft) = 6ft

By Ratio (76): F/5yd = 3ft/1yd; F = 15ft

Liquid Ladder (p.77): 8 \(\cdot\) 2 \(\cdot\) 2 = 32 oz/qt

Linear Ladder (p.77): 12 \(\cdot\) 3 = 36 in/yd

Time Ladder (p.77): 60 \(\cdot\) 24 = 1400 min/day

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