

Word Problems

Extracted from *Algebra Antics* book (pp. 61-78)

The page numbers in this excerpt will *not* match those in *Algebra Antics*.

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Geometric Word Problems

Perimeter Problems

Perimeter [pur-IH-meh-tur] is a measure of the distance around an object. *Peri* is Greek for “around.” *Meter* is Greek for “measure.”

Rectangle

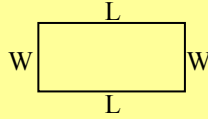
The perimeter of a rectangle is twice its length plus twice its width.

$$P_R = 2L + 2W$$

P_R = Perimeter of rectangle

L = Length (long side)

W = Width (short side)



Alternate Equation: Factoring out the 2 yields: $P_R = 2(L + W)$

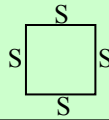
Square

The perimeter of a square is four times the length of one side.

$$P_S = 4S$$

P_S = Perimeter of square

S = Length of one side



Circle

The perimeter, aka circumference [sur-CUM-frens], of a circle is its Diameter times Pi [pii].

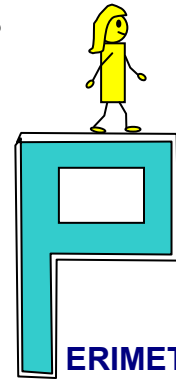
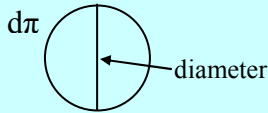
$$C = d\pi$$

C = Circumference

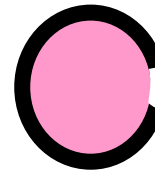
d = diameter (a line through the center)

π = pi = ~ 3.14 or $\sim 22/7$

* The word pi is Greek for periphery and came from measuring circles.

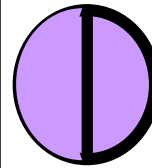


BrainAid
Imagine walking a path around the letter P.



Circumference

BrainAid
Imagine the letter 'C' circling the circumference.



Diameter

BrainAid
Imagine the letter D creating the diameter.

How much fencing is needed to enclose a 100 ft by 50 ft field? (ft = foot or feet)

Identify: Rectangle Perimeter problem

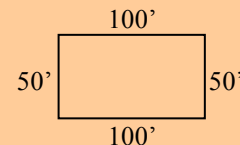
Draw:

Equate: $\text{Perimeter}_{\text{rectangle}} = 2 \cdot \text{length} + 2 \cdot \text{width}$ (all units in feet)

Assign: $P_R = 2L + 2W$

Solve: $P_R = 2(100) + 2(50)$

$P_R = 300 \text{ ft}$



Your turn: What is the distance around a village square that's 30m on each side? (m = meters)

I

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Area Problems

Area [AIR-ee-uh] is a measure of the space on the surface of an object.
Area is Latin for “level ground” or “open space.”

Rectangle

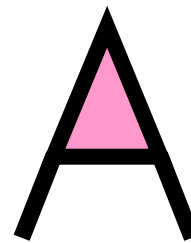
The area of a rectangle is its width times its length.

$$A_R = WL$$

A_R = Area of rectangle

W = Width (short side)

L = Length (long side)



BrainAid
 Imagine the letter 'A' enclosing the area inside its top.

rea

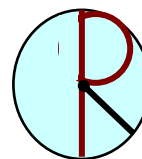
Square

The area of a square is the length of one side squared.

$$A_S = S^2$$

A_S = Area of square

S = Length of one side



adius

BrainAid
 Imagine the slanted leg of the letter 'R' as the radius.

Circle

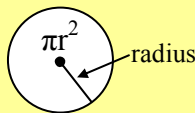
The area of a circle is pi times its radius squared.

$$A_C = \pi r^2$$

A_C = Area of circle

r = radius (a line from center to edge = 1/2 diameter)

π = pi = ~3.14 or ~22/7



How many square feet is a circular lawn whose radius is 10 ft? (ft = foot or feet, ft² = square ft)

Identify: Circle Area problem

Draw:

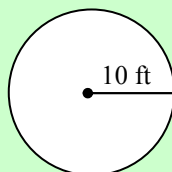
Equate: $Area_{circle} = \pi \cdot \underline{radius}^2$

Assign: $A_C = \pi r^2$

Solve: $A_C = 3.14(10 \text{ ft})^2$

$A_C = 3.14(100 \text{ ft}^2)$

$A_C = \underline{314 \text{ ft}^2}$



Lawn
314 ft²



Seed
157 ft²

How many bags of seed are needed for this lawn if one bag covers 157 ft²?

Identify: Freeform Division problem

Draw:

Equate: $Bags = \frac{Area_{ft^2}}{Coverage_{ft^2}}$

Assign: $B = \frac{A}{C}$

Solve: $B = \frac{314}{157}$

$B = \underline{2 \text{ bags}}$

Your turn: How many square yards is a tarp that measures 50 yd x 30 yd? (yd = yard/s, yd² = square yd)

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Travel Word Problems

Distance/Rate/Time: DRT

The Distance traveled equals the Rate of travel times the Time traveled.

$$D = RT$$

D = Distance traveled

R = Rate of travel (average speed)

T = Time traveled

Travel Rate: $R = D/T$

Travel Time: $T = D/R$

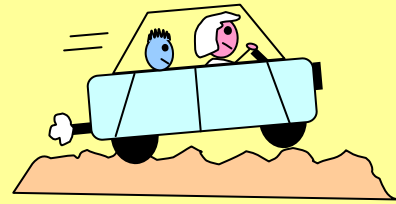
Alternate Equations

$$\frac{D}{T} = \frac{RT}{T}$$

$$\frac{D}{T} = R$$

$$\frac{D}{R} = \frac{RT}{R}$$

$$\frac{D}{R} = T$$



BrainAid

Pronounce DRT as "dirt."
Imagine a car traveling on a dirt road.

What distance is traveled by a biker averaging 10 mph for 2 hours? (mph = miles per hour)

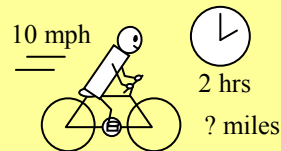
Identify: Travel Distance problem

Draw:

Equate: $\text{Distance}_{\text{mi}} = \text{Rate}_{\text{mph}} \cdot \text{Time}_{\text{hr}}$ (mi=mile/s, hr=hour/s)

Assign: $D = RT$

Solve: $D = 10 \text{ miles/hour} \cdot 2 \text{ hours} = 20 \text{ miles}$



A biker who rides 60 miles in 4 hours pedals how fast on average?

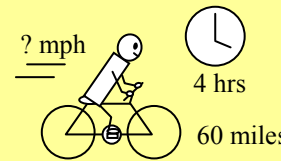
Identify: Travel Rate problem

Draw:

Equate: $\text{Rate}_{\text{mph}} = \text{Distance}_{\text{mi}} / \text{Time}_{\text{hr}}$

Assign: $R = D/T$

Solve: $R = 60 \text{ miles} / 4 \text{ hours} = 15 \text{ mph}$



How long does a biker take to ride 12 miles at 3 mph?

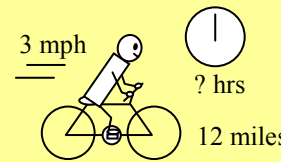
Identify: Travel Time problem

Draw:

Equate: $\text{Time}_{\text{hr}} = \text{Distance}_{\text{mi}} / \text{Rate}_{\text{mph}}$

Assign: $T = D/R$

Solve: $T = 12 \text{ miles} / 3 \text{ miles/hours} = 4 \text{ hours}$



Tip

Instead of memorizing the Alternate Equations, remember $D=RT$, and isolate the unknown variable as needed with Shoot-the-Chute.

$$\frac{D}{T} \leftarrow R$$

$$\frac{D}{R} \leftarrow T$$

R and T always go together.

Your turn: How far does a biker ride when averaging 15 mph for 5 hours?

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Double DRT: Round Trip Average Rate

The average Rate of travel for a round trip (or several shorter trips) is the *total* Distance divided by the *total* Time. $R_{avg} = D_{total} / T_{total}$

What is the average rate of a car that travels 60 miles outbound @ 60 mph, then returns 60 miles inbound @ 30 mph? (mi = mile/s; mph = miles per hour, hr = hour/s)

INCORRECT

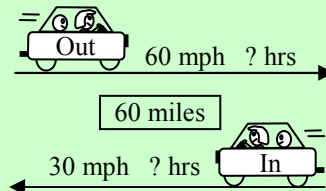
Identify: Average = Sum of Items / Total Items

Draw:

Equate: $R_{avg} = (R_{out} + R_{in}) / 2$

Assign: $R_A = (R_O + R_I) / 2$

Solve: $R_A = (60\text{mph} + 30\text{mph}) / 2 = 90\text{mph} / 2 = 45\text{mph}$



CORRECT

Identify: Travel Time problem (two trips)

Draw:

Equate: $T_{out} = \frac{D_{out}}{R_{out}}$ $T_{in} = \frac{D_{in}}{R_{in}}$

Assign: $T_O = D_O / R_O$

$T_I = D_I / R_I$

Solve: $T_O = 60\text{mi} / 60\text{mph} = 1\text{hr}$

$T_I = 60\text{mi} / 30\text{mph} = 2\text{hr}$

Identify: Round Trip Average Rate

Draw:

Equate: $R_{avg} = \frac{D_{total}}{T_{total}}$

$R_{avg} = \frac{(D_{out} + D_{in})}{(T_{out} + T_{in})}$

Assign: $R_A = (D_O + D_I) / (T_O + T_I)$

Solve: $R_A = (60\text{mi} + 60\text{mi}) / (1\text{hr} + 2\text{hr}) = 120\text{mi} / 3\text{hr} = 40\text{mph}$

Trap!

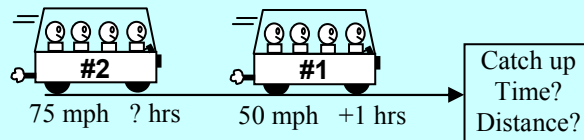
45 mph, the midpoint between 60 mph and 30 mph, would be correct if the car traveled the same amount of time in both directions. But the car necessarily took longer to cover the inbound 60 miles at the slower 30 mph, which pulled the average down to 40 mph.

Your turn: What is the average rate of a car than travels 30 miles outbound @ 30 mph, then returns 30 miles inbound @ 10 mph?

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Double DRT: Catch Up

Bus#1 leaves the depot and averages 50 mph. Bus#2 leaves 1 hour later averaging 75 mph. How long will it take Bus#2 to catch up to Bus#1? At what distance from the depot?



Identify: Travel Catch Up problem (Distance equal)

Draw:

Equate: $\text{Distance}_1 = \text{Distance}_2$ $\text{Time}_1 = \text{Time}_2 + 1 \text{ hr}$ $\text{Rate}_1 = 50\text{mph}$ $\text{Rate}_2 = 75\text{mph}$

Assign: $D_1 = D_2$ $T_1 = T_2 + 1 \text{ hr}$ $R_1 = 50\text{mph}$ $R_2 = 75\text{mph}$

$R_1 T_1 = R_2 T_2$ (substitute)

$R_1(T_2+1) = R_2 T_2$

Solve: $50\text{mph}(T_2+1\text{hr}) = 75\text{mph}(T_2)$

$50T_2\text{mi} + 50\text{mi} = 75T_2\text{mi}$

$\underline{-50T_2\text{mi}} = \underline{-50T_2\text{mi}}$

$\underline{50\text{mi}} = \underline{25T_2\text{mi}}$

$\underline{25\text{mph}} = \underline{25\text{mph}}$

$\underline{2 \text{ hr}} = T_2$

$D_2 = R_2 T_2$

$D_2 = 75\text{mph} \cdot 2\text{hr}$

$D_2 = \underline{150 \text{ mi}}$

Check

$T_1 = T_2 + 1\text{hr}$

$T_1 = 2\text{hr} + 1\text{hr}$

$T_1 = 3 \text{ hr}$

$D_1 = R_1 T_1$

$D_1 = 50\text{mph} \cdot 3\text{hr}$

$D_1 = 150 \text{ mi} \checkmark$

Solution

Bus2 catches up to Bus1 in 2 hours 150 miles from the depot.

Your turn: Ann leaves school and walks 2 mph. Bob leaves 1 hour later and walks 4 mph. How long will it take him to catch up with Ann and at what distance from school?

I

D

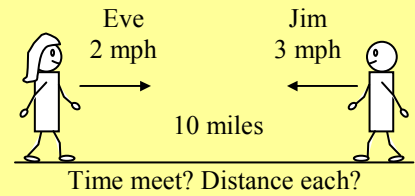
E

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Double DRT: Meet in Between

Eve and Jim are 10 miles apart and start walking towards each other. Eve walks 2 mph. Jim walks 3 mph. How long does it take, and how far has each walked when they meet?



Identify: Travel Meet in Between (Time equal)

Draw:

Equate: $\text{Time}_{\text{Eve}} = \text{Time}_{\text{Jim}} = \text{Time}$ $\text{Distance}_{\text{Eve}} + \text{Distance}_{\text{Jim}} = 10 \text{ mi}$ $\text{Rate}_{\text{Eve}} = 2 \text{ mph}$ $\text{Rate}_{\text{Jim}} = 3 \text{ mph}$

Assign: $T_E = T_J = T$ $D_E + D_J = 10 \text{ mi}$ $R_E = 2 \text{ mph}$ $R_J = 3 \text{ mph}$

Add equations to save steps

$$\begin{aligned} D_E &= R_E T \\ + D_J &= R_J T \\ \hline (D_E + D_J) &= (R_E + R_J)(T) \end{aligned}$$

(substitute)

Why it works to add equations

$$\begin{aligned} 3 &= 3 \\ + 4 &= 4 \\ \hline 3 + 4 &= 3 + 4 \end{aligned}$$

Solution

They meet in 2 hours. Ann walked 4 miles; Bob walked 6 miles.

Solve: $10 \text{ mi} = (2+3) \text{ mph}(T)$

$D_E = R_E T$

$D_J = R_J T$

$$\frac{10 \text{ mi}}{5 \text{ mph}} = \frac{5 \text{ mph}(T)}{5 \text{ mph}}$$

$D_E = 2 \text{ mph} \cdot 2 \text{ hr}$

$D_J = 3 \text{ mph} \cdot 2 \text{ hr}$

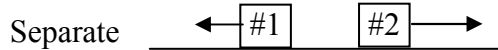
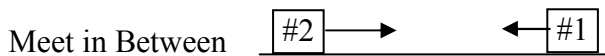
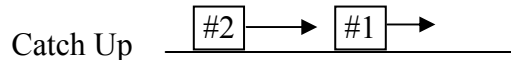
$2 \text{ hr} = T$

$D_E = 4 \text{ mi}$

$D_J = 6 \text{ mi}$

Double DRT Variations

Many variations of distance, rate, and time and the relationships between them are possible, e.g., travelers may leave at same or different times, total distance may be provided but not individual distances, rates may be given in terms of each other as in "twice as fast." The variations seem endless but are always based on $D = RT$.



Tip

See Word Problem Analysis for the minimum elements needed to solve 2Eq6Unk problems like Double DRTs.

Mileage: DMV

Distance traveled equals Mileage rate times fuel Volume.

$D = MV$

D = Distance in miles

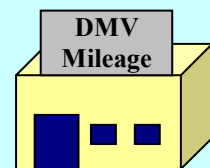
M = Mileage in miles per gallon (mpg)

V = Volume of fuel in gallons (gal)

Alternate Equations

$M = D/V$

$V = D/M$



BrainAid

DMV stands for the Dept. of Motor Vehicles.

A car travels 500 miles on a 20-gallon tank of gas. What is its mpg?

Identify: Travel Mileage



Draw:

Equate: $\text{mpg} = \frac{\text{Distance}_{\text{mi}}}{\text{Volume}_{\text{gal}}}$

Assign: $M = D/V$

Solve: $M = 500 \text{ mi} / 20 \text{ gal} = 25 \text{ mpg}$

Financial Word Problems

Besides financial items, these equations will work for almost any type of percent increase, decrease, or change problems. Tip: Review percents, fractions, and decimals in *Max Learning's Fraction Fun*.

Price Markup on Cost: NO+MO

Merchants mark up (raise) the price of a product so they can make a profit on each sale.

The New price equals the Old price plus the Markup% times the Old price.

$$N = O + MO$$

N = New price (aka Retail or List price)

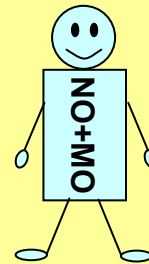
O = Old price (aka Wholesale or Original price)

M = Markup Percent

Alternate Equation: Factoring out the O yields: $N = O(1 + M)$.

Explanation: $(1 + M)$ is the multiplier that yields the New price, e.g., if Markup = 20%, the New price is 120% of the Old.

The math: $(1 + 20\%) = (100\% + 20\%) = 120\%$



BrainAid

Imagine a positive (+) merchant named NO+MO [noh-moh] who loves to Markup prices.

What is the price of a \$10 coat after a 20% markup?

Identify: Price Markup problem

Draw:

Equate: $\text{New price} = \text{Old price} + \text{Markup}\% \cdot \text{Old price}$

Assign: $N = O + MO$

Solve: $N = \$10 + 20\%(\$10) = \$10 + \$2 = \text{\$12}$



Markup on Sell Price

Some merchants prefer to make markups based on the selling price, which results in a greater new price. They use this equation instead:

$$N = O + MN$$

Your turn: What is the price of a \$20 coat after a 50% markup?

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Tax "Markup": NO+TO

Adding sales tax to an item is like marking it up by the tax percentage.

$$N = O + TO$$

T = Tax percentage (replaces M)

Imagine a NO+MO has a cousin named NO+TO [noh-toh].

What is the price of a \$12 coat with 5% sales tax?

Identify: Tax Markup problem

Draw:

Equate: $\text{New price} = \text{Old price} + \text{Tax}\% \cdot \text{Old price}$

Assign: $N = O + TO$

Solve: $N = \$12 + 5\%(\$12) = \$12 + \$0.60 = \text{\$12.60}$



Tip

To distinguish between the New markup price and the New price after tax, use different subscripts for N, e.g.,

N_M = New markup price

N_T = New price after tax

Price Discount: NO-DO

Merchants discount (lower) the price of a product to increase the number of items sold.

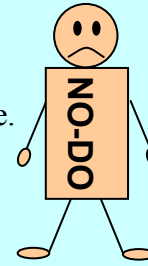
The New price equals the Old price minus the Discount% times the Old price.

$$N = O - DO$$

N = New price (aka Discounted or Sale price)

O = Old price (aka Retail, List, or Original price)

D = Discount Percent



BrainAid

Imagine a negative (-) merchant named NO-DO [noh-doh] who has to Discount prices.

Alternate Equation: Factoring out the O yields: $N = O(1 - D)$.

Explanation: $(1 - D)$ is the multiplier that yields the New price, e.g., if Discount = 30%, the New price is 70% of the Old.

The math: $(1 - 30\%) = (100\% - 30\%) = 70\%$

What is the price of a \$10 T-shirt after a 25% discount?

Identify: Price Discount problem

Draw:

Equate: $\text{New price} = \text{Old price} - \text{Discount}\% \cdot \text{Old price}$

Assign: $N = O - DO$

Solve: $N = \$10 - 25\%(\$10) = \$10 - \$2.50 = \text{\$7.50}$



Shortcut Solution

If 25% is deducted,
75% remains.

$$75\%(\$10) = \$7.50$$

Your turn: What is the price of a \$20 T-shirt after a 50% discount?

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Your turn: What is the price of the discounted T-shirt (from above) with 10% sales tax?

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Percent-Change: PN-O/O

Merchants sometimes need to calculate the percent change between two prices to determine the markup or discount percent.

The Percent change equals the difference between the New price and the Old price divided by the Old price.

$$P = \frac{N - O}{O}$$

P = Percent change

N = New price (aka Current price)

O = Old price (aka Original or Base price)



BrainAid

Imagine PiNocchio, the puppet who became a boy, has the nickname PNOO [pi-noh]. He calculates the percent change in his nose size when he tells a lie.

$$Pi = \frac{N - O}{cchiO}$$

Markup-to-%Change Proof

$$\begin{aligned} N &= O + MO \\ \frac{N}{O} &= \frac{O + MO}{O} \\ \frac{N - O}{O} &= \frac{MO}{O} \\ \frac{N - O}{O} &= M \end{aligned}$$

(% Markup)

Discount-to-%Change Proof

$$\begin{aligned} N &= O - DO \\ \frac{N}{O} &= \frac{O - DO}{O} \\ \frac{N - O}{O} &= \frac{-DO}{O} \\ \frac{N - O}{O} &= -D \end{aligned}$$

(% Discount)

We could move the minus sign to the left side to make $(-N + O) / O = D$, but then you'd have to learn two equations. It's easier to use the same equation and remember that a negative result means a percent decrease or discount.

What is the percent markup on a \$10 dress that now sells for \$15?

Identify: Percent Change problem

Draw:

Equate: $\text{Percent-change} = (\text{New price} - \text{Old price}) / \text{Old price}$

Assign: $P = (N - O) / O$

Solve: $P = (\$15 - \$10) / \$10 = 5/10 = 50\%$



What is the percent discount on a \$15 dress that's on sale for \$10?

Identify: Percent Change problem

Draw:

Equate: $\text{Percent-change} = (\text{New price} - \text{Old price}) / \text{Old price}$

Assign: $P = (N - O) / O$

Solve: $P = (\$10 - \$15) / \$15 = -5/15 = -33\%$



The minus indicates a decrease. Because a discount is also a decrease, we'd say the percent discount is 33%, not -33%.

Percent Paradox

A larger Old price results in a smaller percent change.

Although the difference between the New and Old prices was \$5 for both markup and discount, the percent changes were *not* the same, because they were based on different Old prices, first \$10 (50% change) then \$15 (-33% change).

Your turn: What is the percent discount on a \$20 dress that's on sale for \$15?

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Cost: CPK

Merchants must often calculate the cost of selling or buying a quantity of identical items.

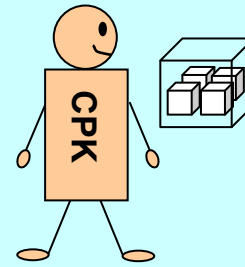
The Cost equals the Price per item times the Kwantity of items.

$$C = PK$$

C = Cost of all items

P = Price (aka Cost) for one item

K = Kwantity (made-up word) of items purchased



BrainAid

Imagine a merchant named CPK [see-pak] seeing a package of items he sold.

What is the total cost of 5 hammers sold for \$10 each?

Identify: Cost problem

Draw:

Equate: Cost of hammers = price per hammer • kwantity of hammers

Assign: C = PK

Solve: $C = \$10/\cancel{\text{hammer}} (5 \cancel{\text{ hammers}}) = \text{\$50}$



If a box contained twenty \$10 hammers, how much would 3 boxes cost?

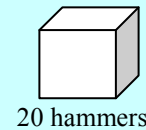
Identify: Cost problem (1 of 2)

Draw:

Equate: Cost of box = price per hammer • kwantity of hammers per box

Assign: C = PK

Solve: $C = \$10/\cancel{\text{hammer}} (20 \cancel{\text{ hammers}}/\text{box}) = \$200/\text{box}$



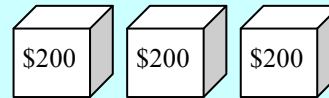
Identify: Cost problem (2 of 2)

Draw:

Equate: Cost of 3 boxes = price per box • kwantity of boxes

Assign: C = PK

Solve: $C = \$200/\cancel{\text{box}} (3 \cancel{\text{ boxes}}) = \text{\$600}$



Your turn: What is the total cost of 4 toasters sold for \$15 each?

I

D

E

A

S

Also see **Unit Analysis** and **Proportional Ratios** for alternative approaches to setting up and solving Cost and other problems.

Interest Earned: IRP

Interest earned on an investment equals the Rate (percent) of annual interest times the Principal invested.

$$I = RP$$

I = Interest earned (\$)

R = Rate of annual interest (percent)

P = Principal invested (\$)



BrainAid

Imagine Mr. IRP who proudly exclaims: "I ReaP interest from my investments."

How much interest does Ron earn in one year on a \$1000 investment at 6%?

Identify: Interest Earned problem

Draw:

Equate: Interest earned = interest rate • principal

\$1000 6% \$Interest earned?

The traditional equation is $I=PR$, but $I=RP$ fits the $Q=PK$ pattern.

Assign: $I = 6\% \cdot \$1000$

Solve: $I = 6\% \cdot \$1000 = 6/100 \cdot \$1000 = \text{\$60}$

I = RPT

T = Time period (in years or fraction of a year)

The traditional equation is $I=PRT$.

$I=RP$ is derived from $I=RPT$ where $T=1$. But any period can be used, e.g., $T=2$ equals 2 years. $T=1/12$ equals one month.

BrainAid: Mr. IRPT exclaims: "I reap interest RePeaTedly" over several periods.

$I = RPT$ computes *simple* interest, which is calculated only on the originally invested Principal each period.

FYI: For *compound* interest, add the interest earned each period to the Principal, then compute the next period's interest on the new higher total. Compounding is a good thing for the investor since it increases the total interest earned.

Coins: TVC

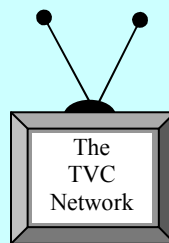
The Total value of a group of the same-type coin is the Value of one coin times the number of those Coins.

$$T = VC$$

T = Total value

V = Value of one coin of that type

C = Coins of that type



BrainAid

Imagine TVC, the Total Value Channel, having a special sale on valuable coins.

What is the total value of Ned's 30 nickels?

$T = \$.05/\text{nickel} \cdot 30 \text{ nickels} = \1.50

Coin problems usually combine the total values of several coin equations.

- The variable 'C' changes for each coin type: P=Pennies, N=Nickels, D=Dimes, Q=Quarters.
- Total $\text{All coins} = \$.01P + \$.05N + \$.10D + \$.25Q$

Peg has a total of 7 dimes and quarters worth \$1. How many of each does she have?

Identify: Coin problem

Draw:

7 coins: Dimes? Quarters?

\$1

Equate: Dimes + Quarters = 7 Total value = \$.10 • Dimes + \$.25 • Quarters

Assign: $D + Q = 7$ $\$1.00 = \$.10D + \$.25Q$

Solve: $D - Q = -Q$ $\$1.00 = \$.10(7-Q) + \$.25Q$

$D = 7-Q$ $\$1.00 = \$.70 - \$.10Q + \$.25Q$

(substitute) $\$1.00 = \$.70 + \$.15Q$

$D = 7-2$ $-\$.70 \quad -\$.70$

$D = 5$

$\$.30 = \$.15Q$

$\$.15 = \$.15$

$2 = Q$

Solution

Peg has 5 dimes and 2 quarters.

Check

$\$1.00 = \$.10D + \$.25Q$
 $\$1.00 = \$.10(5) + \$.25(2)$
 $\$1.00 = \$.50 + \$.50$
 $\$1.00 = \$1.00 \checkmark$

Work Word Problems

Work/Rate/Time: WRT

The Work completed equals the Rate of work times the Time worked.

$$W = RT$$

W = Work completed (aka job, task)

R = Rate of work

T = Time worked

Alternate Equations: $R = W/T$, $T = W/R$

WRT problems are very similar to DRT problems (p.5), except the "distance" traveled is the work completed.



BrainAid
Imagine WRT the Work Rate Timekeeper keeping track of how far workers have gone towards completing the work.

How many tasks can Rob complete if he performs 1 task in 2 hours and works for 10 hours?

$$W = 1 \text{ task}/2 \text{ hours} \cdot 10 \text{ hours} = 5 \text{ tasks}$$

Work problems usually combine the work rates of more than one worker.

Cal can paint 1 room in 2 hours. Zoe can paint 1 room in 3 hours.

How long does it take them to paint 1 room together?

Identify: Work problem

Draw:

$$\text{Equate: } \text{Work}_{\text{Both}} = (\text{Rate}_{\text{Cal}} + \text{Rate}_{\text{Zoe}}) \text{ Time}$$

$$\text{Assign: } W = (R_C + R_Z) T$$

$$\text{Solve: } 1 \text{rm} = (1 \text{rm}/2 \text{hr} + 1 \text{rm}/3 \text{hr}) T$$

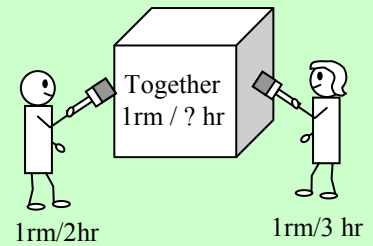
$$6[1 \text{rm}] = (1 \text{rm}/2 \text{hr} + 1 \text{rm}/3 \text{hr}) T] \leftarrow \text{Clear Denominators}$$

$$6 \text{rm} = (3 \text{rm}/\text{hr} + 2 \text{rm}/\text{hr}) T$$

$$\frac{6 \text{rm}}{5 \text{rm}/\text{hr}} = \frac{(5 \text{rm}/\text{hr}) T}{5 \text{rm}/\text{hr}}$$

$$1 \frac{1}{5} \text{ hr} = T$$

rm = room/s
hr = hour/s



Your turn: Ona can assemble one bike in 1 hour. Mac can assemble one bike in 2 hours.

How long does it take for them to assemble one bike together?

I

D

E

A

S

Mixture Word Problems

Volume/Amount/Total: VAT

The Volume of one component is an Amount (percent) of the Total mixture.

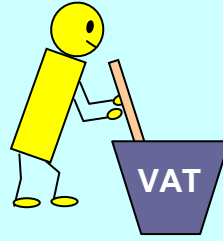
$$V = AT$$

V = Volume of one component

A = Amount (percent)

T = Total volume of mixture

Alternate Equations: $A = V/T$, $T = V/A$



BrainAid
Imagine stirring a mixture in a large VAT.

How many ounces (oz) of water are in a 100 oz beaker that's 25% water?

$$V = 25\%(100 \text{ oz}) = 25 \text{ oz}$$

Typical mixture problems involve changes to the volume of components.

Ethanol is 40% of a 10-pint mixture. Tim adds 2 more pints of ethanol. What is its new percent?

Identify: Mixture problem

Draw:

$$\begin{aligned} \text{Equate: } \frac{\text{Volume of ethanol}}{\text{Volume ethanol}+2} &= \frac{\text{Amount \% ethanol}}{\text{Amount \% ethanol}+2} \cdot \frac{\text{Total volume}}{\text{Total volume}+2} \end{aligned}$$

$$\text{Assign: } V_E = A_E T$$

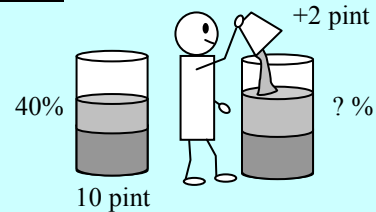
$$V_{E2} = A_{E2} T_2$$

$$\text{Solve: } V_E = 40\%(10\text{pt}) = 4\text{pt} \rightarrow (4+2\text{pt}) = A_{E2}(10+2\text{pt})$$

$$\text{pt} = \text{pint/s}$$

$$\frac{6\cancel{\text{pt}}}{12\cancel{\text{pt}}} = \frac{A_{E2}(12\cancel{\text{pt}})}{12\cancel{\text{pt}}}$$

$$\textcircled{50\%} = A_{E2}$$



Your turn: Methyl is 10% of a 100-gallon mixture. Kai adds 20 more gallons of methyl. What is its new percent?

I

D

E

A

S

Conversion Word Problems

A word problem may require you to convert one unit of measure into another.

Conversions: NCO

The number of New units equals the Conversion rate times the number of Old units.

$$N = CO$$

N = New units

C = Conversion rate

O = Old units

Alternate equations: $C = N/O$; $O = N/C$

The variables for N and O will change depending on the units being converted.

Rae walked for 1.5 hours. How many minutes did she walk?

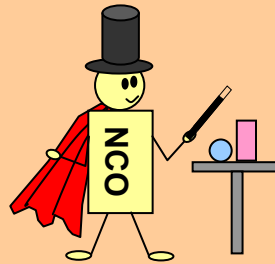
Identify: Conversion problem

Draw:

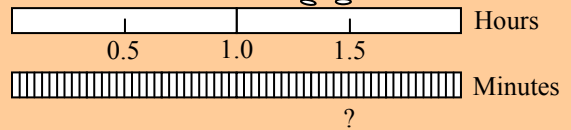
Equate: Minutes walked = 60 minutes per hour • hours walked

Assign: $M = 60\text{min/hr} \cdot H$ (min = minute/s; hr = hour/s)

Solve: $M = 60\text{min/hr} \cdot 1.5\text{hr} = \textcircled{90 \text{ min}}$



BrainAid
Imagine NCO
[nuu-coh] the
magician
making New
Conversions.



Nat bought a 96-inch piece of wood. How many feet is it?

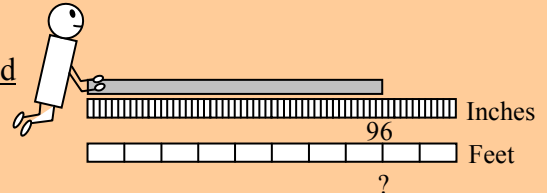
Identify: Conversion problem

Draw:

Equate: Feet of wood = 1 foot per 12 inches • inches of wood

Assign: $F = 1\text{ft}/12\text{in} \cdot I$ (ft = foot/feet; in = inch/es)

Solve: $F = 1\text{ft}/12\text{in} \cdot 96\text{in} = \textcircled{8 \text{ ft}}$



Your turn: Eve has a 10-foot tree in her yard. How many inches tall is it?

I

D

E

A

S

Conversion by Replacement/Ratio

Most conversions are usually part of a more complicated word problem and don't always merit the full IDEAS treatment. Below are two alternate conversion methods.

Conversion by Replacement

Replace the old unit with its equivalent in the new unit and multiply.

3 hours = ? minutes

Process: Replace "hours" with "60 minutes" and multiply.

Solution: 3 hours = 3(60 minutes) = 180 minutes

24 inches = ? feet

Process: Replace "inches" with "1/12 foot" and multiply.

Solution: 24 inches = 24(1/12 foot) = 2 feet

Why Replacement Works:

It's based on $N=CO$ being reversed to $OC=N$.

Old units • Conversion rate = New units

3 ~~hours~~ • 60 minutes/~~hour~~ = 180 minutes

24 ~~inches~~ • 1/12 foot/~~inch~~ = 2 feet

Your turn: 120 min = ? hr

Tip: 1 min = 1/60 hr

Your turn: 2 yds = ? ft

Conversion by Ratio

Set the New/Old ratio to the Conversion-rate ratio.

Solve for the New unit. Tip: Use Shoot-the-Chute.

How many seconds (S) are in 10 minutes?

$$\frac{S}{10 \text{ min}} = \frac{60 \text{ sec}}{1 \text{ min}}$$

$$\frac{S}{10 \text{ min}} \xrightarrow{\text{cancel}} \frac{60 \text{ sec}}{1 \text{ min}}$$

$$S = \frac{(10 \cancel{\text{min}})(60 \text{ sec})}{1 \cancel{\text{min}}}$$

$$S = \underline{600 \text{ sec}}$$

Why Ratios Work:

They're based on $N=CO$ being altered to $N/O = C$.

Inverse Ratios

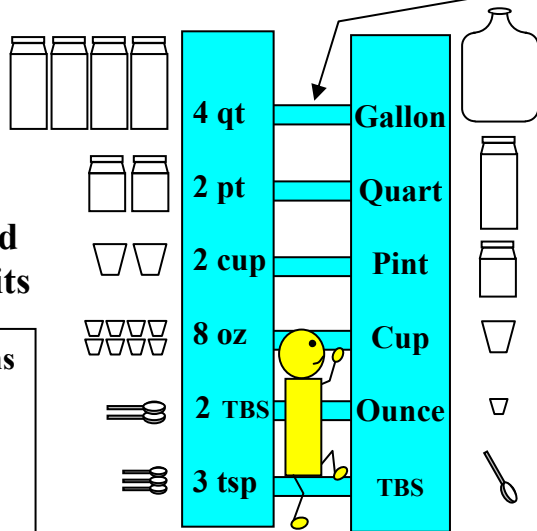
In problems that place the unknown variable in the denominator of the ratio, make sure the units in the conversion-rate ratio match top to bottom. See Unit Analysis.

Your turn: How many feet (F) are in 5 yards?

Conversion Ladders

Ladders make multistep conversions easy!

U.S. Liquid Ladder



Equivalents
Each ladder rung links equivalent units (e.g., 4 qt = 1 Gallon) and yields a Conversion rate (e.g., $C = 4 \text{ qt/Gallon}$).

Multistep Conversions

- Start at the Old unit on the left side.
- Multiply up the left side until you reach the New unit's rung.

Water Weight

1 pt = 1 lb
1 qt = 2 lb
1 gal = 8 lb
pt-pint
lb-pound/s
gal-gallon

tsp-teaspoon; TBS-tablespoon
oz-ounce; pt-pint; qt-quart

How many teaspoons in an ounce?

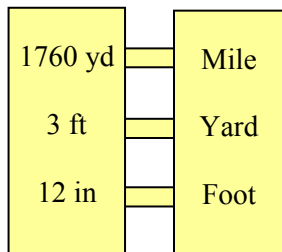
Procedure: Start at 3 tsp. Climb/multiply to 2 TBS.

Solution: $3 \cdot 2 = 6$

Why it works: $3 \text{ tsp/TBS} \cdot 2 \text{ TBS/oz} = 6 \text{ tsp/oz}$

Your turn: How many ounces in a quart?

U.S. Linear Ladder



in-inch/es
ft-foot/feet; yd-yard/s

How many feet in a mile?

$3 \cdot 1760 = 5280$

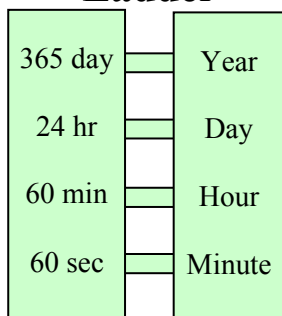
Your turn: How many inches in a yard?

Tip

Larger dictionaries often contain conversion charts that compare U.S. to Metric units of measure.

Look under "measure" or in the appendix.

Time Ladder



sec-second/s
min-minute/s; hr-hour/s

How many seconds in a day?

$60 \cdot 60 \cdot 24 = 86400$

Your turn: How many minutes in a day?

Trap!

Some conversion charts list conversion equations like:

miles \times 1.6 = kilometers

This does *not* mean that
1.6 miles = 1 kilometer.

It means $OC = N$ where
O = Old units = miles

C = Conversion rate = 1.6 km/mi
N = New units = kilometers

In fact,
1 mile = 1.6 kilometers.

Answer Key

Page Error! Bookmark not defined.: Freeform Word Problems

1EqUnk: $Meg_{plums} = 6 - 2 + 4 = 8$.

2EqUnk: $Bob_{pens} = 2 + Jan_{pens}$; $Bob_{pens} + Jan_{pens} = 10$. $Bob_{pens} = 6$; $Jan_{pens} = 4$.

Page 3: Geometric Word Problems

Perimeter (p.3): $Perimeter_{square} = 4 \cdot Side = 4(30) = 120$ m

Area (p.4): $Area_{rect} = Length \cdot Width = 50 \cdot 30 = 150$ yd²

Page 5: Travel Problems

DRT (p.5): $Distance = 15 \text{ miles/hour} \cdot 5 \text{ hours} = 75$ miles

Double DRT: Round Trip Average Rate (p.6): The average rate for the round trip was 15 mph.

$$T_{out} = D_O/R_O = 30 \text{ mi} / 30 \text{ mph} = 1 \text{ hr}; T_{in} = D_I/R_I = 30 \text{ mi} / 10 \text{ mph} = 3 \text{ hr}$$

$$R_{avg} = (D_O + D_I) / (T_O + T_I) = (30 \text{ mi} + 30 \text{ mi}) / (1 \text{ hr} + 3 \text{ hr}) = 60 \text{ mi} / 4 \text{ hr} = 15 \text{ mph}$$

Double DRT: Catch up (p.7): Bob catches up to Ann in 1 hour 4 miles from school.

$$D_{Ann} = D_{Bob}; T_A = T_B + 1; R_A(T_B + 1) = R_B T_B; 2 \text{ mph}(T_B + 1) = 4 \text{ mph}(T_B); T_B = 1 \text{ hr}; D_B = 4 \text{ mph} \cdot 1 \text{ hr} = 4 \text{ mi}$$

Page 9: Financial Word Problems

Price Markup on Cost (p.9): $N = \$20 + 50\%(\$20) = \$20 + \$10 = \$30$

Price Discount (p.10): $N = \$20 - 50\%(\$20) = \$20 - \$10 = \$10$; $N_{tax} = \$10 + 10\%(\$10) = \$10 + \$1 = \$11$

Percent-Change (p.11): $P = (\$15 - \$20) / \$20 = -5/20 = -25/100 = -25\%$

Cost (p.12): $C = \$15/\text{toaster} \cdot 4 \text{ toasters} = \60

Page 14: Work Word Problems

WRT: $W_{Both} = (R_{Ona} + R_{Mac})T_{Both}$; $1 \text{ bk} = (1 \text{ bk/hr} + 1 \text{ bk/2hr})T$; $T = 2/3 \text{ hr}$ (bk=bike)

Page 15: Mixture Word Problems

VAT: $V_M = 10\%(100 \text{ gal}) = 10 \text{ gal}$. $(10 + 20 \text{ gal}) = A_{M20}(100 + 20 \text{ gal})$. $A_{M20} = 25\%$

Page 16: Conversion Word Problems

NCO (p.16): $Inches = 12 \text{ in/ft} \cdot 10 \text{ ft} = 120$ inches

By Replacement (p.17): $120 \text{ min}(1/60 \text{ hr}) = 2 \text{ hr}$. $2 \text{ yd}(3 \text{ ft}) = 6 \text{ ft}$

By Ratio (p.17): $F/5 \text{ yd} = 3 \text{ ft}/1 \text{ yd}$; $F = 15 \text{ ft}$

Liquid Ladder (p.18): $8 \cdot 2 \cdot 2 = 32$ oz/qt

Linear Ladder (p.18): $12 \cdot 3 = 36$ in/yd

Time Ladder (p.18): $60 \cdot 24 = 1400$ min/day