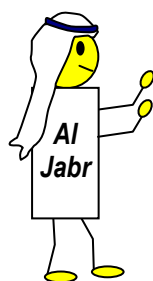


Problem Types

Freeform: You create the equations.
Standard: You use predefined equations.



Bring Together
Word problems describe relationships of quantities. Your goal is to bring known and unknown quantities together into equations, then solve for the unknowns.
Algebra = Al Jabr = bring together

Freeform
Reflecting the almost infinite ways of relating quantities, *freeform* problems require you to convert words to math. You decide which variables to use and how to relate them in equations.

Standard
For problems that embody common relationships, mathematicians have created *standard* equations. You write out the equation, plug in the given values, and solve for the unknowns.

Freeform Problems

Creating equations by converting words to math.

Generic Freeform Problems

Last month, Sara ate 10 apples. This month she ate 3 more than twice as many. How many did she eat?

Equation: $A = 2(10) + 3$

A notebook costs \$1.00 more than a pencil. Together they cost \$1.20.

What is the price of the pencil? (Trap! The answer is *not* 20 cents.)

Equations: $N = P + 1.00$ $N + P = 1.20$

Age Freeform Problems

Pete is 3 years older than Jean. Together their ages add to 21. How old is Jean?

Equations: $P = J + 3$ $P + J = 21$

In 5 years, Meg will be twice Nat's age. Last year she was 15 years older. How old is Meg?

Equations: $(M + 5) = 2(N + 5)$ $(M - 1) = (N - 1) + 15$

Numerical Freeform Problems

• Number

Find these numbers: The first is 5 less than twice the second. Their product is 88.

Equations: $F = 2S - 5$ $FS = 88$

35 is the sum of which two consecutive numbers?

Equation: $n + (n+1) = 35$

• Digit

Find the number whose tens digit is twice its units digit and whose sum of digits is 12.

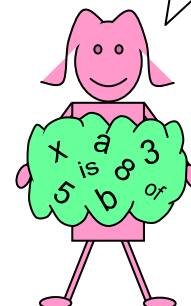
Equations: $t = 2u$ $t + u = 12$

• Digit-Value

The units digit of a 2-digit number is 3 times the tens digit. If the digits are reversed, the sum of the original and reversed numbers is 132. What is the original number?

Equations: $u = 3t$ $(10t + u) + (10u + t) = 132$

I will freely form this jumble into an equation.



Which Variables to Use?

It's common to use X for the unknown variable. But letters that relate to the items they represent make more sense.

You must be flexible, however, since the same letter can represent different items in different problems. For example, P can be Price, Percent, Principal, etc.

You'll find solutions and strategies for these and each problem type at MaxLearning.Net

Proportion Problems

Comparing *ratios* of related items.

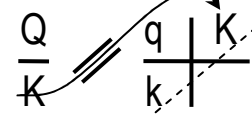
Q = Quantity (unknown)
 K = Kwantity (given)
 q = quantity (given)
 k = kwantity (given)

$$\frac{Q}{K} = \frac{q}{k}$$

[kwik-kwik]

Kwantity & kwantity
 are made-up words.

Shoot-the-Chute



Q & q have the same units.
 K & k have the same units.

K and k units
 dissolve,
 leaving only
 Q = q units.

- Given K, q, & k, set up and solve for Q using Shoot-the-Chute.
- For relations of 3 or more items, use a Q/K = q/uac/k chain.
- Q/K=q/k is the parent of Conversion, Q=RK, and Q=PK problems.

Q/K=q/k (2 items)

A recipe that feeds 4 boys calls for 2 cups of flour. How many cups are needed to feed 6 boys?

$$\text{Equation: } \frac{Q \text{ cups}}{6 \text{ boys}} = \frac{2 \text{ cups}}{4 \text{ boys}}$$

Three boxes hold 18 pens. How many pens do 7 boxes hold?

$$\text{Equation: } \frac{Q \text{ pens}}{7 \text{ boxes}} = \frac{18 \text{ pens}}{3 \text{ boxes}}$$

$$\frac{Q}{K} = \frac{q}{k}$$

Q/K=q/uac/k chain (3 or more items)

Three boxes hold 18 pens. Two cases contain 20 boxes. How many pens are in 5 cases?

$$\text{Equation: } \frac{Q \text{ pens}}{5 \text{ cases}} = \frac{18 \text{ pens}}{3 \text{ boxes}} \left| \frac{20 \text{ boxes}}{2 \text{ cases}} \right. \leftarrow \text{Extended Fraction Fence}$$

Conversion Problems

Converting *units* of measure.

Q/K=q/k Conversion

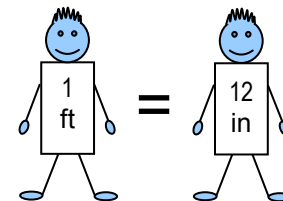
How many feet are in 30 inches?

$$\text{Equation: } \frac{Q \text{ feet}}{30 \text{ inch}} = \frac{1 \text{ foot}}{12 \text{ inch}}$$

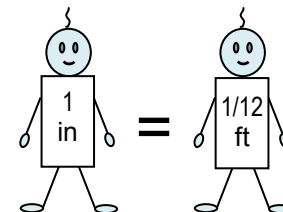
Conversion Twin Magic Hat Trick

How many feet are in 30 inches?

$$\text{Computation: } 30 \text{ in} = 30 (1 \text{ in}) = 30 (1/12 \text{ ft})$$



Conversion Twins



Conversion Twin Stack Attack

How many feet are in 30 inches?

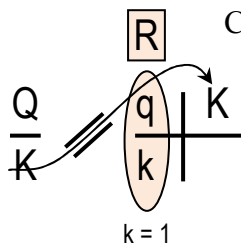
$$\text{Computation: } \frac{30 \text{ in}}{12 \text{ in}} \left| \frac{1 \text{ ft}}{1} \right.$$

Conversion Twin Chain (3 or more units of measure)

How many inches are in 3 miles?

$$\text{Computation: } \frac{3 \text{ mi}}{1 \text{ mi}} \left| \frac{5280 \text{ ft}}{1 \text{ ft}} \right| \frac{12 \text{ in}}{1 \text{ in}}$$

Rate Problems



Comparing items whose *ratio* forms a third item.

$$Q = RK$$

kyu-raak

Quantity = Rate • Quantity

$$Q = \frac{R}{1} K$$

- A rate is a ratio.
- A unit rate has a denominator of 1.
- $Q=RK$ is derived from $Q/K=q/k$.
- R is the ratio of q/k where $k=1$.
- R is treated as a 3rd unit of measure.

Travel Problems: $D=RT$ [dirt]

Distance = Rate • Time

Joey drives a dirt road at 25 mph. How far does he travel in 3.75 hours?

Equivalencies: $R = 25$ $T = 3.75$

Average Rate: $R_{Avg} = (D_1 + D_2) / (T_1 + T_2)$

Sara drives 15 miles uphill at 30 mph and returns downhill at 60 mph. What is her average speed?

Equivalencies: $D_U = 15$ $D_D = 15$ $R_U = 30$ $R_D = 60$ (Trap! The answer is *not* 45 mph.)

Catch Up: $D_1 = D_2$

A bus leaves the station averaging 50 mph. One hour later a car leaves to pursue the bus. If the car averages 70 mph, how long does it take to catch up to the bus? Equivalencies: $D_B = D_C$ $R_B = 50$ $R_C = 70$ $T_B = T_C + 1$

Meet in Between: $D_1 + D_2 = D_{Total}$

A train departs City A averaging 80 mph towards City B, which is 340 miles away. At the same time, a train departs City B averaging 90 mph. When do they meet in between? Equivalencies: $R_A = 80$ $R_B = 90$ $T_A = T_B$ $D_T = 340$

Move Apart: $D_1 + D_2 = D_{Total}$

From the same airport at the same time, one plane heads West at 500 mph while another heads East at 600 mph. How far have they moved apart after 2.5 hours? Equivalencies: $R_W = 500$ $R_E = 600$ $T_W = 2.5$ $T_E = 2.5$

Work Problems: $W=RT$ [wart]

Work = Rate • Time

Tim builds 3.5 frames per day. How many can he build in 11 days? Equivalencies: $R = 3.5$ $T = 11$

Combined Work: $W_1 + W_2 = W_{Total}$

Ann's garden hose can fill her pool in 4 days. Her neighbor Bea's larger hose can fill the same pool in

2 days. How many days will it take for both hoses to fill the pool? Equivalencies: $R_A = \frac{1}{4}$ $R_B = \frac{1}{2}$ $T_A = T_B$ $W_T = 1$

Wage = Rate • Time

At \$23.50 per hour, how much does Noah earn in 8 hours? Equivalencies: $R = 23.50$ $T = 8$

Cost Problems: $C=PN$ [coupon]

Cost = Price/unit • Number of units

What is the cost of 5 candy bars priced at \$1.25 each? Equivalencies: $P = 1.25$ $N = 5$

Cost Mixture: $C_1 + C_2 = C_{Mix}$

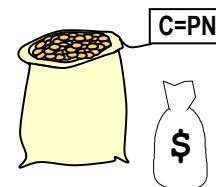
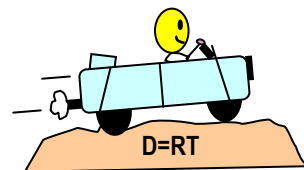
Sam mixes 2 lbs. of \$4/lb of apple drops with 3 lbs. of \$5/lb banana chips.

How much should he charge for the mixed candy? Equivalencies: $P_A = 4$ $P_B = 5$ $N_A = 2$ $N_B = 3$

Coin Problems: $C_1 + C_2 = C_{Total}$

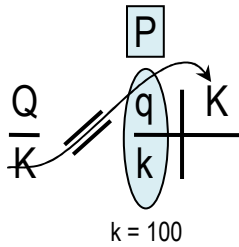
Tom has 10 coins consisting of quarters and dimes worth \$1.45. How many of each coin does he have?

Equivalencies: $P_Q = .25$ $P_D = .10$ $N_Q + N_D = 10$ $C_T = 1.45$



Percent Problems

Comparing items where one is *part* of another.



$$Q = PK$$

kyu-pik

Quantity = Percent • Kwantity

- A percent is a ratio with denominator 100.
- $Q=PK$ is derived from $Q/K=q/k$.
- P is the ratio of q/k where $k=100$.
- When $P > 100\%$: $Q = K + PK$ [kwik-pik], which is the basis for NOMO, NODO, and PNOO problems.

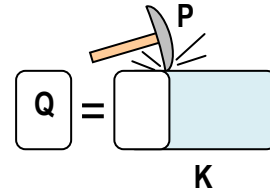
Percent-Of Problems: $Q=PK$ [kyu-pik]

Quantity = Percent • Kwantity

What is 20% of 400? Equivalencies: $P = 20\%$ $K = 400$

80 is what percent of 400? Equivalencies: $Q = 80$ $K = 400$

80 is 20% of what? Equivalencies: $Q = 80$ $P = 20\%$

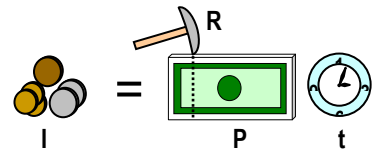


Interest Problems: $I=RPt$ [I reaped]

Interest = Rate • Principal • time

How much interest is earned on \$450 at 5% for 3 years?

Equivalencies: $R = 5\%$ $P = 450$ $t = 3$



Mixture Problems: $V=AT$ [vat]

Volume = Amount • Total

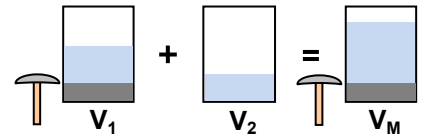
What is the volume of alcohol in a 16 oz solution that is 35% alcohol?

Equivalencies: $A = 35\%$ $T = 16$

Percent Mixture: $V_1 + V_2 = V_{\text{Mix}}$

How much pure water will dilute 5 gallons of salt water from 15% to 10% salt?

Equivalencies: $A_1 = 15\%$ $T_1 = 5$ $A_2 = 0\%$ $A_M = 10\%$

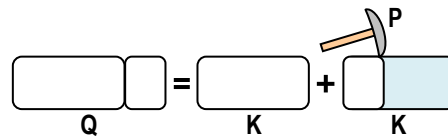


Markup Problems: $N=O+MO$ [noh-moh]

New = Old + Markup • Old

What is the new price of a \$30 dress marked up 25%?

Equivalencies: $O = 30$ $M = 25\%$

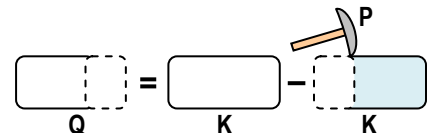


Discount Problems: $N=O-DO$ [noh-doh]

New = Old - Discount • Old

What was the original price of a suit on sale for \$140 after a 30% discount?

Equivalencies: $N = 140$ $D = 30\%$



Percent-Change Problems: $P=(N-O)/O$ [pih-noh-kee-oh]

Percent Change = (New - Old) / Old

What is the percent change of a population that increased from 200 to 250?

Equivalencies: $N = 250$ $O = 200$

Solutions at **MaxLearning.Net**

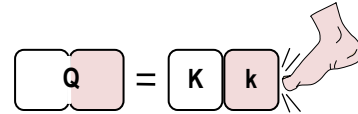
Blended-Unit Problems

Comparing items whose *product* forms a third item.

- Q is a combination of K and k units.
- K and k have no denominators, so no units are dissolved.
- $Q_{\text{UNIT-unit}} = K_{\text{UNIT}} \bullet k_{\text{unit}}$

$$Q = Kk$$

kyu-kik



$$\underline{Q} \text{ quantity} = \underline{K} \text{ quantity} \bullet \underline{k} \text{ quantity}$$

Worker-Time Problems: $Q_{W-T} = K_W \bullet k_T$

Five employees work 4 hours on a job. How many worker-hours did the job take?

Equivalencies: $K_W = 5$ $k_T = 4$

How long would it take for 3 machines to complete a job that takes 12 machine-days? (Machines are "workers.")

Equivalencies: $Q_{W-T} = 12$ $K_W = 3$

Two workers dig 4 holes in 15 minutes. How many holes can 6 workers dig in 30 minutes?

Equivalencies: $K_{W1} = 2$ $k_{T1} = 15$ $K_{W2} = 6$ $k_{T2} = 30$ (Tip: Use $Q/K=q/k$ to relate holes to worker-minutes.)

Person-Time Problems: $Q_{P-T} = K_P \bullet k_T$

Four people have enough water to last 6 days. If 2 more people arrive, how long will the water last?

Equivalencies: $K_{P1} = 4$ $k_{T1} = 6$ $K_{P2} = K_{P1} + 2$

If 2 people eat 3 lbs of food in 4 days, how much food will 3 people eat in 8 days?

Equivalencies: $K_{P1} = 2$ $k_{T1} = 4$ $K_{P2} = 3$ $k_{T2} = 8$ (Tip: Use $Q/K=q/k$ to relate food to people-days.)

Solutions at
MaxLearning.Net

Multi-part problems often require a mix of standard and/or freeform equations.

Other Problem Types

Hundreds of additional problem types exist, some with complex standard formulas. But most involve the four basic operations ($+$ $-$ \bullet $/$) with variables in direct or inverse proportion.

- Geometry (shape measurements: perimeter, area, volume...)
- Statistics (mean, median, mode, range...)
- Probability (likelihood of an outcome, odds...)
- Trigonometry (triangle relationships: sin, cos, tan...)
- Calculus (areas under curves, rates of change of curve, math of motion...)
- Science (physics, chemistry, biology...)
- Engineering (civil, electrical, mechanical...)
- Construction (measurements, quantities...)
- Manufacturing (measurements, processes...)
- Financial (accounting, investments...)

You'll find word problems for

- Geometry
- Statistics
- Probability

under those categories at MaxLearning.Net





Your Turn



Fill in the abbreviation that best identifies each problem type.

FF: Freeform PR: Proportion CO: Conversion RA: Rate PE: Percent BU: Blended-Unit

- 1 ____ What is 35% of 560?
- 2 ____ How long will it take 3 laborers to complete a 15 worker-hour task?
- 3 ____ Ann is 3 years older than Bob. Their ages add to 17. How old is Bob?
- 4 ____ How many liters are in 5 gallons of gasoline?
- 5 ____ Tom travels 50 miles in 1.5 hours. How fast is he driving?
- 6 ____ If 2 wagons carry 8 people, how many people can 7 wagons carry?
- 7 ____ How much oil should be added to 3 quarts of a 10% solution to make it 25% oil?
- 8 ____ What is the price per pound if 15 pounds of carrots cost \$22.50?
- 9 ____ If the tens digit is 1/3 of the units digit, and the digits add to 8, what is the number?
- 10 ____ What is the price of a \$95 dress that is being sold for 25% off?

Match each problem type with its equation.

- | | |
|------------------------|--------------------|
| 11 ____ Proportion | A. $W = RT$ |
| 12 ____ Rate | B. $N = O + MO$ |
| 13 ____ Percent | C. $V = AT$ |
| 14 ____ Work | D. $C = PN$ |
| 15 ____ Cost | E. $Q/K = q/k$ |
| 16 ____ Travel | F. $I = RPt$ |
| 17 ____ Interest | G. $P = (N - O)/O$ |
| 18 ____ Mixture | H. $Q = PK$ |
| 19 ____ Markup | I. $D = RT$ |
| 20 ____ Discount | J. $N = O - DO$ |
| 21 ____ Percent-Change | K. $Q = RK$ |

Answers: [1] PE [2] BU [3] FF [4] CO [5] RA [6] PR [7] PE [8] RA [9] FF [10] PE
[11] E [12] K [13] H [14] A [15] D [16] I [17] F [18] C [19] B [20] J [21] G