Proportion Problems

Comparing *ratios* of related items.

Q = Quantity (unknown)
K = Kwantity (given)
q = quantity (given)
k = kwantity (given)

Kwantity & kwantity are made-up words.

$$\frac{Q}{K} = \frac{q}{k}$$

[kwik-kwik]

Shoot-the-Chute

$$\frac{Q}{K} \quad \frac{q}{k} \quad \frac{K}{1}$$

K and k units dissolve, leaving only Q = q units.

Q & q have the same units.
K & k have the same units.

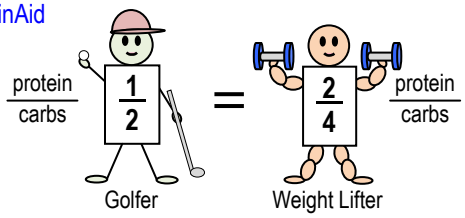
- Given K, q, & k, set up and solve for Q using Shoot-the-Chute.
- For relations of 3 or more items, use a Q/K = q/uac/k chain.
- Q/K=q/k is the parent of Conversion, Q=RK, and Q=PK problems.

Equivalent Pro Portions

BrainAid

RATIO
relation
fraction

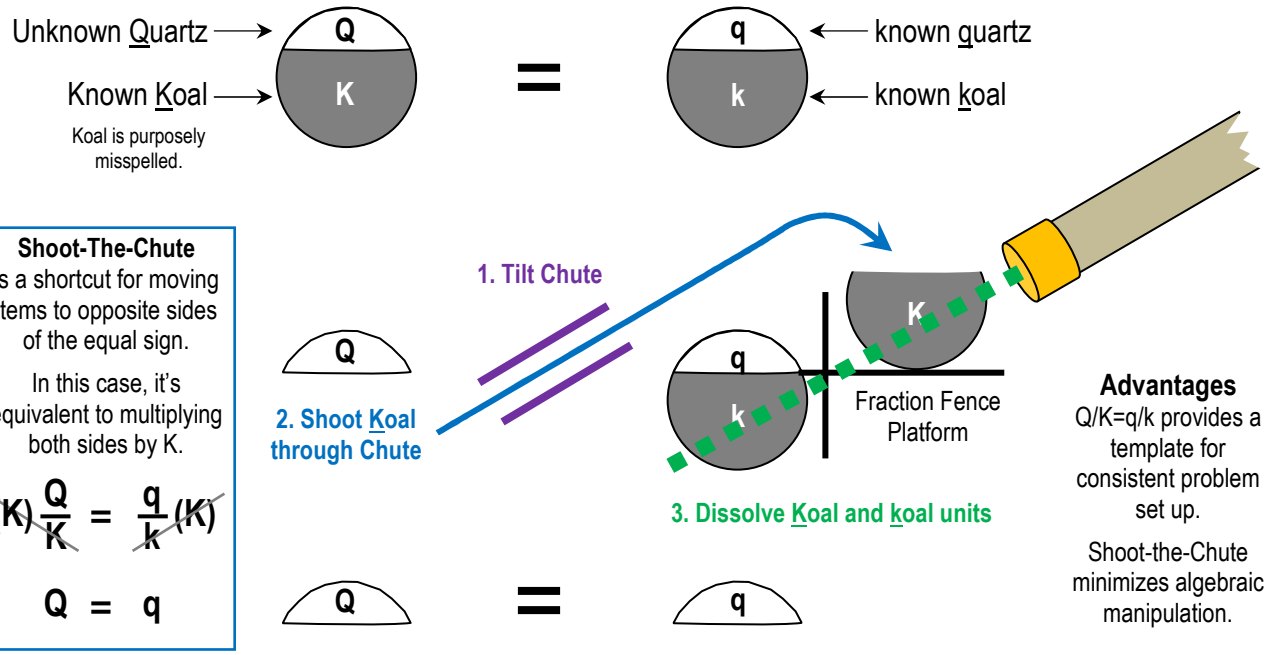
PROPORTION
ratio = ratio



EQUIVALENT
VALUE
Equal in value, not in appearance.

Imagine that pro athletes eat rations of nutrients in the same pro portions: 1 part protein to 2 parts carbs. Athletes in light sports need fewer calories so eat smaller portions than those in heavy sports.

Shoot-the-Chute



Shoot-The-Chute
is a shortcut for moving items to opposite sides of the equal sign.

In this case, it's equivalent to multiplying both sides by K.

$$(K)\frac{Q}{K} = \frac{q}{k}(K)$$

$$Q = q$$

Advantages
Q/K=q/k provides a template for consistent problem set up.

Shoot-the-Chute minimizes algebraic manipulation.

Q/K = q/k

Relations of 2 items [kwik-kwik]

$$\frac{Q}{K} = \frac{q}{k}$$ ← Fraction Fence

It takes 2 cups to feed 4 kids. How many cups are needed to feed 6 kids?

Set up

Items
1) cups
2) kids

Find cups to feed 6 kids...

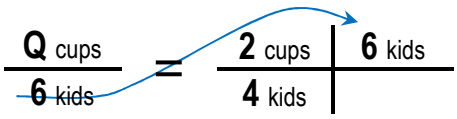
$$\frac{Q \text{ cups}}{6 \text{ kids}} = \frac{2 \text{ cups}}{4 \text{ kids}}$$

...given that 2 cups feed 4 kids.

Observe "cups" on top, "kids" on bottom.

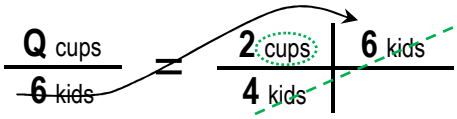
Shoot-the-Chute

Draw the "shoot" line *through* "6 kids" to show it's gone.



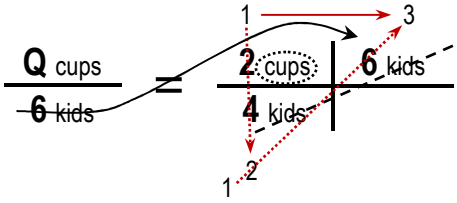
"6 kids" is now on the Fraction Fence.

Dissolve Units



Dissolve "kids" units. Only "cups" remain, which is the goal.

Do the Math



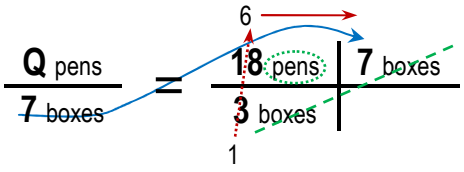
Divide vertically and/or diagonally when possible, then multiply across.

Q = 3 cups

Circle or box the solution.

If three boxes hold 18 pens, how many pens do 7 boxes hold?

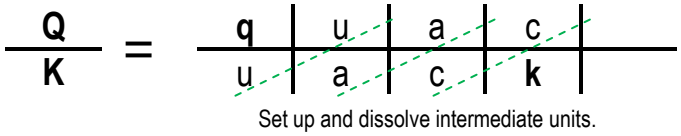
Items
1) pens
2) boxes



Q = 42 pens

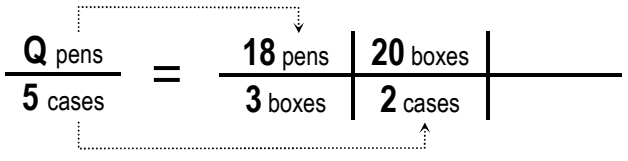
Q/K = q/uac/k

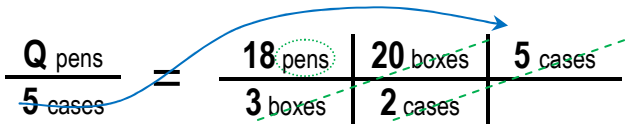
Relations of 3 or more items [kwik-kwak]

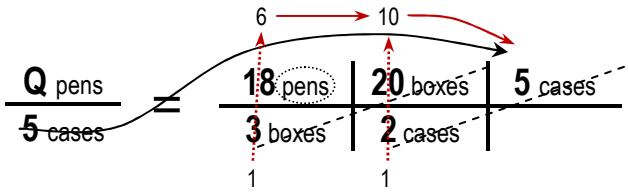


If three boxes hold 18 pens and two cases contain 20 boxes, how many pens are in 5 cases?

Items
1) pens
2) boxes
3) cases



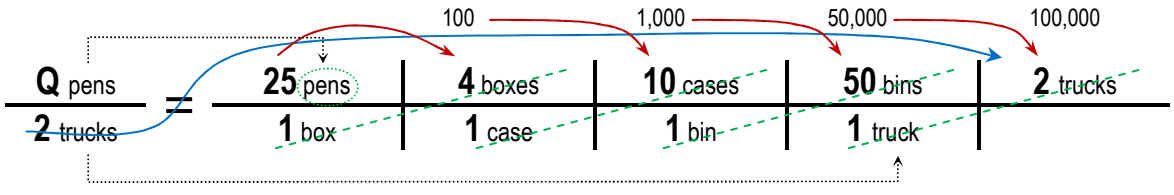




Q = 300 pens

How many pens are in 2 trucks if there are 25 pens per box, 4 boxes per case, 10 cases per bin, and 50 bins per truck?

Items
1) pens
2) boxes
3) cases
4) bins
5) trucks



Q = 100,000 pens

Proportion Problem Traps

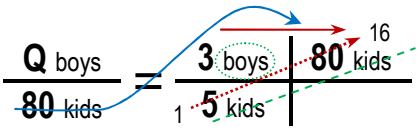
Proportion Total Trap

A private school enrolls 3 boys for every 5 girls.
If there are 80 kids, how many are boys? How many are girls?

WRONG!

$$\frac{Q \text{ boys}}{80 \text{ kids}} = \frac{3 \text{ boys}}{5 \text{ kids}}$$

Trap: 5 = girls, *not* total kids.



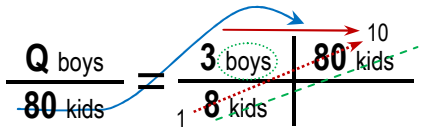
$$Q = 48 \text{ boys}$$
$$80 - 48 = 32 \text{ girls}$$

48:32 ≠ 3:5
There should be *fewer* boys than girls!

CORRECT

$$\frac{Q \text{ boys}}{80 \text{ kids}} = \frac{3 \text{ boys}}{8 \text{ kids}}$$

Calculate total kids from the 3:5 ratio:
3 boys + 5 girls = 8 kids



$$Q = 30 \text{ boys}$$
$$80 - 30 = 50 \text{ girls}$$

30:50 = 3:5
The *correct* ratio of boys to girls.

Inversely Proportional Traps

A biker rides at an average rate of 4 miles per hour (mph) for 8 hours.
What rate of speed must he ride to cover the *same* distance in 2 hours?

WRONG!

$$\frac{Q \text{ mph}}{2 \text{ hrs}} = \frac{4 \text{ mph}}{8 \text{ hrs}}$$

Trap: mph and hrs are *inversely* proportional.

$$Q = 1 \text{ mph}$$

He must ride *faster* to cover the same distance in less time!

CORRECT

$D_1 = D_2$
$R_1T_1 = R_2T_2$
$4(8) = R_2(2)$
$16 \text{ mph} = R_2$

See the Q=RK Rate Problems lesson for further explanation.

EXPLANATION

Q/K = q/k works for items that are *directly* proportional.

In Q=RK problems, **R**ate and **K**wantity are *inversely* proportional. For a fixed **Q**uantity, as **R**ate increases, **K**wantity decreases, and vice versa.

Q = RK

Directly Proportional
- **Q**uantity & **R**ate
- **Q**uantity & **K**wantity

Solve as Q/K=q/k or Q=RK.

Inversely Proportional
- **R**ate & **K**wantity

Solve as Q=RK.

See the Problem Analysis lesson for further explanation.

Note: In Q/K=q/k, Q is the unknown. In this problem, Q=Rate. In Q=RK, Q is the *product* of Rate & Kwantity.

10% of a 40 oz rock is gold. What percent of an 80 oz rock would yield the same amount of gold?

WRONG!

$$\frac{Q \%}{80 \text{ oz}} = \frac{10 \%}{40 \text{ oz}}$$

Trap: gold % and rock size are *inversely* proportional.

$$Q = 20 \%$$

For a larger rock, a *smaller* % of it would yield the same gold.

CORRECT

$Q_1 = Q_2$
$P_1K_1 = P_2K_2$
$10\%(40) = P_2(80)$
$5\% = P_2$

See the Q=PK Percent Problems lesson for further explanation.

EXPLANATION

Q/K = q/k works for items that are *directly* proportional.

In Q=PK problems, **P**ercent & **K**wantity are *inversely* proportional. For a fixed **Q**uantity, as **P**ercent increases, **K**wantity decreases, and vice versa.

Q = PK

Directly Proportional
- **Q**uantity & **P**ercent
- **Q**uantity & **K**wantity

Solve as Q/K=q/k or Q=PK.

Inversely Proportional
- **P**ercent & **K**wantity

Solve as Q=PK.

See the Problem Analysis lesson for further explanation.

Note: In Q/K=q/k, Q is the unknown. In this problem, Q=Percent. In Q=PK, Q is the *product* of Percent & Kwantity.

Your Turn!

True or False

1) _____ A ratio is a relation between numbers.

2) _____ Equivalent ratios form a proportion.

3) _____ With Q/K=q/k, you set up and solve proportions the same way each time.

4) _____ Shoot-the-Chute reduces the steps needed to isolate the unknown variable.

5) _____ Q/K=q/k works for inversely proportional items.

6) A jar of instant coffee makes 20 cups. How many cups would 4 jars make?

7) A jar of instant coffee makes 20 cups. How many jars are needed to make 60 cups?

8) A pack of gum has 10 sticks. A box has 20 packs. How many sticks are in 4 boxes?

9) A pack has 10 sticks; a box has 20 packs; a crate has 15 boxes. How many sticks are in 5 crates?

10) For every 4 apples, John picks 3 oranges. If he picks 70 pieces of fruit, how many are apples?

Answers: 1T, 2T, 3T, 4T, 5F, 6) 80 cups, 7) 3 jars, 8) 800 sticks, 9) 15000 sticks, 10) 40 apples