

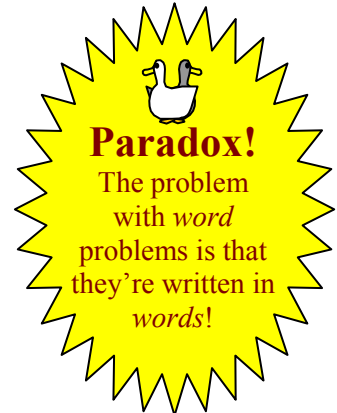
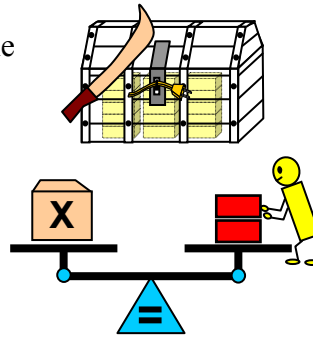


Worry Problems!



Worry (aka word-story) Problems cause grief to students and teachers alike. But once you learn the secrets to uncovering their hidden treasure (the answer), they can be great fun!

Worry Problems use algebra **equations** (like a balance scale) to compare unknown values to known values to find what's inside the **variable** box *without opening it!*



Words to Math!

Translating vague words into precise math symbols is challenging because of the tremendous number of ways that the same thing can be said with different words. To complicate the task, some words, like *of*, can have completely *opposite* meanings depending on how they are used!

This table translates some of the key words you'll encounter and shows the resulting variables and equations.

Ann is the same age as Bob. is <u>Ann's</u> age equals <u>Bob's</u> age. $A = B$	=	Carl and Don are the same height. are <u>Carl's</u> height equals <u>Don's</u> height. $C = D$
Ellen has 2 more toys than Frank. more <u>Ellen's</u> toys equal <u>Frank's</u> toys plus 2 more. $E = F + 2$	+	George is 3 years older than Hanna. older <u>George's</u> age equals <u>Hanna's</u> age plus 3 years. $G = H + 3$
Ida's time was 1 hour less than Jack's. less <u>Ida's</u> time equals <u>Jack's</u> time minus 1 hour. $I = J - 1$	-	Kelly is 4 years younger than Lance. younger <u>Kelly's</u> age equals <u>Lance's</u> age minus 4 years. $K = L - 4$
Gravity on Earth is 6 times the Moon's. times <u>Earth's</u> gravity is 6 times the <u>Moon's</u> gravity. $E = 6M$	×	Mel worked 1/2 of 4 hours. 1/2 of 4 <u>Mel's</u> hours were one half times 4 hours. $M = \frac{1}{2}(4)$
What is the cost of one toy at \$24 per dozen? per One <u>Toy</u> costs \$24/12. $T = 24/12$	÷	<u>Nancy</u> ate 3 of 4 pieces of pie. 3 of 4 <u>Nancy's</u> portion was 3 pieces divided by 4 pieces. $N = 3/4$

Variables traditionally are written on the left, e.g., $E = F + 2$ is preferred to $E = 2 + F$.

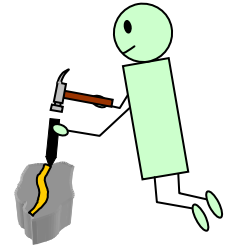
IDEAS for Worry Problems



Algorithm
Use IDEAS to take the worry out of word-story problems!



BrainAid: Imagine that a word problem is a murky stream filled with rocks (equations). Some of these rocks contain streaks of gold (unknown values) that require tools (known values, equalities, substitutions) to extract. You'll need creativity and your best IDEAS to extract the gold!



IDEAS	Explanation	Example
<u>I</u>dentify	<p>Identify the problem type as:</p> <ul style="list-style-type: none"> <i>Freeform</i>: you create the equation/s. <i>Standard</i>: you use existing equations. 	<p>What distance is traveled by a biker averaging 10 mph for 2 hours?</p> <p>Type: Standard travel</p>
<u>D</u>raw	<p>Draw simple pictures or symbols of the items in the problem. Label values and units of measure. This will help you “see” beyond the often confusing words.</p>	<p>? miles 2 hrs</p>
<u>E</u>quate	<p>Equate the given and unknown items into an equation (rock), using meaningful variables and correct units of measure.</p>	<p>D = distance traveled (miles). R = rate of travel (miles/hour). T = time traveled (hours).</p> <p>D = R T</p>
<u>A</u>cquire	<p>Acquire given values, equalities, and substitutions (tools) from the problem, including units of measure.</p>	<p>R = 10 <u>miles</u> / hour T = 2 hours</p>
<u>S</u>olve	<p>Solve for the unknown value/s using the <i>WAC Golden Plug</i> algorithm to extract the gold (solution).</p> <ul style="list-style-type: none"> Include units of measure so the numbers have meaning. Omit units from variables. Be sure the unwanted units dissolve leaving the proper units behind. Keep items vertically aligned (like rain) so you don't omit any terms. Use the <i>Conversion Twins</i> technique to convert units of measure as needed. Circle the final answer/s. Check your answer/s. 	<p>Solve</p> $D = R T$ $D = 10 \frac{\text{miles}}{\text{hour}} (2 \text{ hours})$ $D = 20 \text{ miles}$ <p>Check</p> $D = R T$ $20 = 10(2)$ $20 = 20 \checkmark$

IDEAS steps can often be combined for more efficient problem solving. Simpler problems generally don't require every IDEAS step.

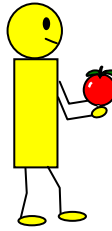
FreeForm Problems

FreeForm problems have no underlying equations, so require Words-to-Math conversions.

1 Equation / 1 Unknown (1EqUnk)

To be solvable, a word problem must provide one equation with one unknown 1EqUnk [WUN-ek-unk] or provide enough information to reduce more complex equations to 1EqUnks.

UNSOLVABLE
1 Equation / 2 Unknowns
 John had 5 apples. He gave some away.
 How many did he have left?
Apples left equal 5 minus Gave away.
 $A = 5 - G$
 $A = ?$



SOLVABLE
1 Equation / 1 Unknown
 John had 5 apples. He gave 2 away.
 How many did he have left?
Apples left equal 5 minus 2 gave away.
 $A = 5 - 2$
 $A = 3$

2 Equations / 2 Unknowns (2 EqUnk)

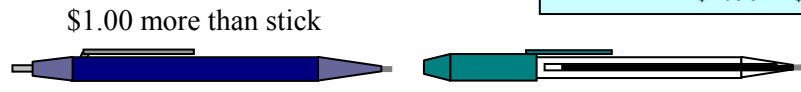
If the word problem yields 2EqUnks [TU-ek-unks] (aka Simultaneous Equations), one of the variables must be eliminated (by addition or subtraction) or replaced with the other variable (by substitution).

A ballpoint pen costs \$1.00 more than a stick pen. Together both cost \$1.20. What is the cost of the stick pen? The ballpoint?

Trap!
 The stick pen does *not* cost 20 cents.

Alternatively, this problem could be set up as a 1EqUnk:
 $x = \text{cost of stick pen}$
 $x + \$1.00 = \text{cost of ballpoint}$
 $x + x + \$1.00 = \1.20

Identify: FreeForm 2EqUnk



Draw:

$B = \text{\$cost of ballpoint} + S = \text{\$cost of stick pen} = \$1.20$

Equate: $B = S + \$1.00$

$B + S = \$1.20$

Acquire:

Solve:

SumMore Technique

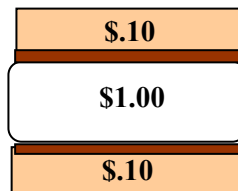
Problems like the above that have a "Sum" and a "More" can be solved without algebra.

An actual S'more, made over a campfire, consists of a melted marshmallow between chocolate & graham-cracker wafers.

Sum of \$1.20 is the entire S'more

More of \$1.00 is the Marshmallow.

Each wafer is $\frac{1}{2}$ of the Sum minus the More.
 $\frac{1}{2}(\$1.20 - \$1.00) = \$0.10$



The cheaper stick pen is one wafer of \$.10

The more expensive ballpoint pen is the marshmallow plus the other wafer.
 $\$1.00 + \$0.10 = \$1.10$

Conversion Problems



Worry Problems sometimes require items to be converted from one unit of measure to an equivalent unit of measure. You could set up proportions or use a conversion factor, but it's easy to fall into the trap of reversing the results using these methods. Instead, use the Conversion Twins Magic Hat Algorithm.

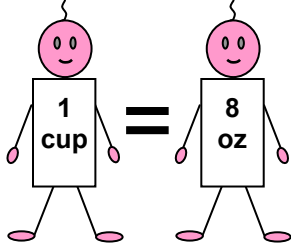


Algorithm
Use Conversion Twins and the Magic Hat to convert units of measure!

A standard cup holds 8 ounces (oz) of liquid.
5 cups contain how many ounces?

1. Create Twin Brothers

Draw the unit to be converted as a 1-unit twin.

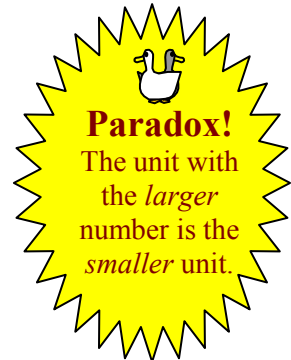
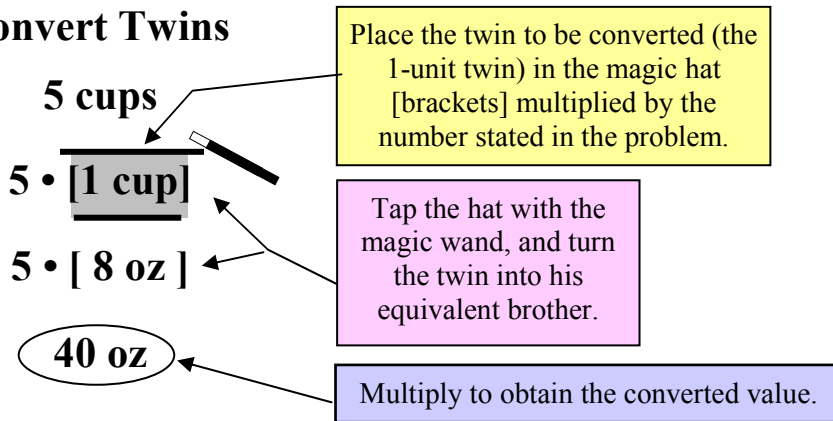


Draw the equivalent value twin.

BrainAid
Imagine identical twins with equal values but different names!

TIP!
Look in a dictionary under MEASURES for equivalents.

2. Convert Twins



48 ounces fills how many cups?

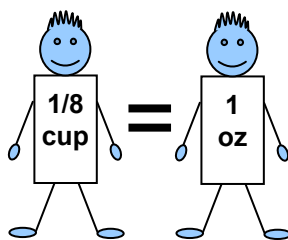
To Create Twin Cousins

Divide both twin brothers by the value of the non-1-unit brother.

$$1 \text{ cup} = 8 \text{ oz}$$

$$\frac{1 \text{ cup}}{8} = \frac{8 \text{ oz}}{8}$$

$$\frac{1 \text{ cup}}{8} = 1 \text{ oz}$$



BrainAid
Imagine the twin brothers have cousins who are also twins!

48 oz

$$48 \cdot [\quad]$$

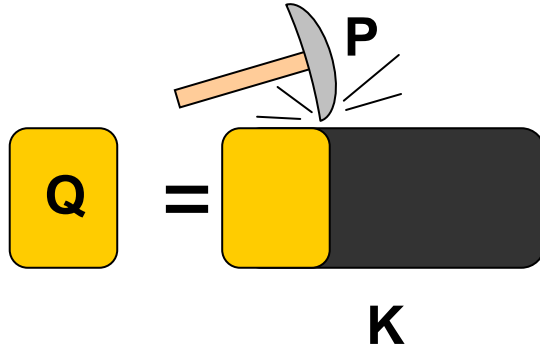
$$48 \cdot [\quad]$$


_____ cups

Q=PK Problems

Q=PK [kyu-pik] is the generic format for problems that involve one (Q)uantity as a (P)ercentage of another (K)uantity. Although Q and K represent different items, they are measured in the *same* units. P is a percentage, which has no units.


Kuantity is purposely misspelled to differentiate it from Quantity.

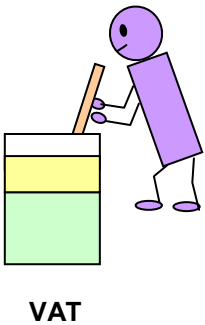


 **BrainAid**
Imagine using a prospector's pick to separate a (Q)uantity of gold as a (P)ercentage of the whole roc(K).

Q=PK Problems

- Mixture: $V = AT$
Volume of component = Amount% • Total mixture.
- Interest: $I = RP$: Interest earned = Rate% • Principal
The traditional equation reverses RP, and adds Time: $I = PRT$

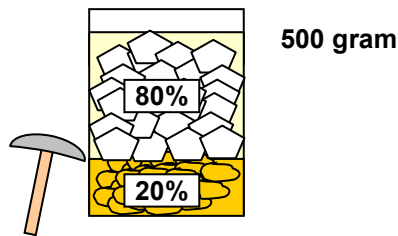
 **BrainAid**
Imagine stirring a mixture in a VAT where the (V)olume of the desired component is a percentage (A)mount of the (T)otal mixture.



Q-Pick Pete mixes a 500 gram vat that contains a slurry of 20% gold and 80% quartz. How many grams of gold are in the vat? How many grams of quartz?

Identify: Mixture

Draw:



V_q = Volume of quartz (grams)
 V_g = Volume of gold (grams)

Equate:

$$V_g = A_g T$$

$$V_q = A_q T$$

Acquire:

Solve:

Q=RK Problems

Q=RK [kyu-rawk] problems solve relationships between two quantities.

Q=RK problems involve *two* proportionally-related items which have *different* units:

Quantity & Kwantity (purposely misspelled!).

Q=RK is a linear equation in the form $y=mx+b$ where $y=Q$, $m=q/k$, and $b=0$.

R is *not* a third quantity. It's a ratio of q/k units.

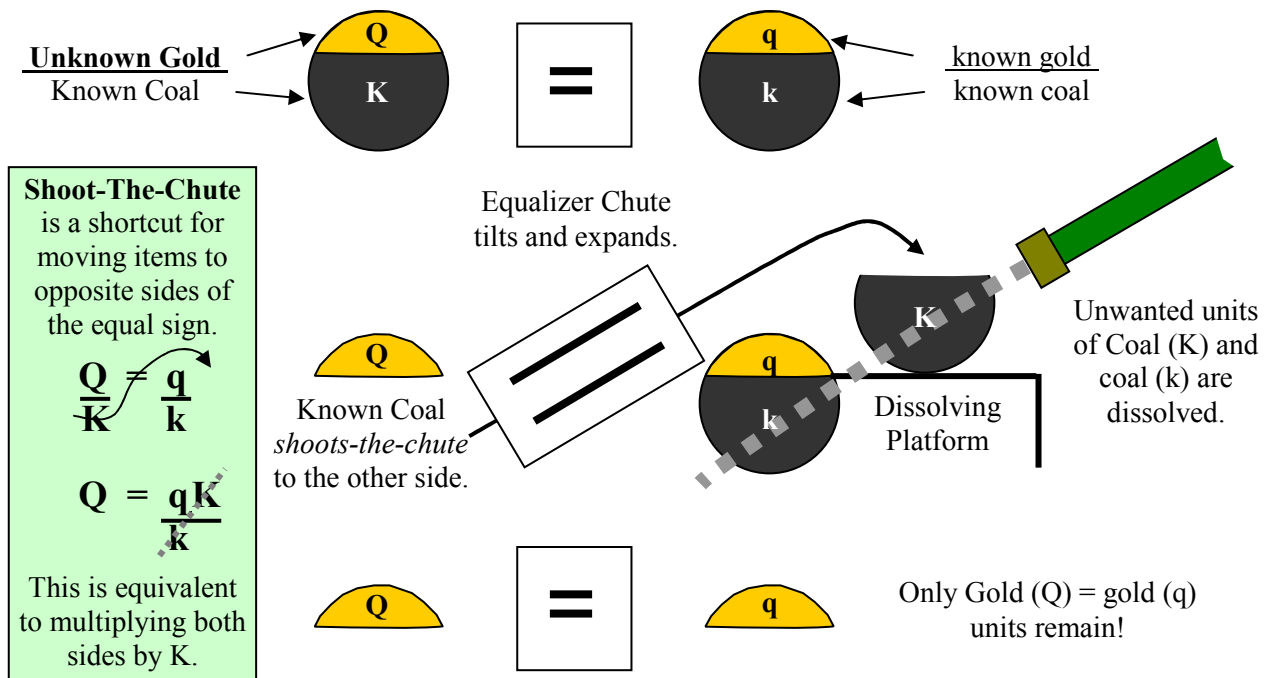
$$\begin{aligned} Q &= R K \\ Q &= \frac{q}{k} K \\ Q &= q \end{aligned}$$

K and k units dissolve, leaving only Q = q units.

Q-Rock Gold Mining Method

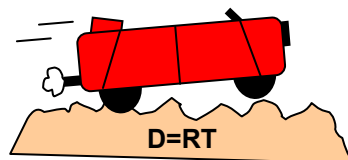
Q=RK problems can be manipulated algebraically to solve for R or K, but setting them up as proportional ratios makes it easier to isolate any desired variable. Q and q are gold; K and k are coal.

$$\frac{Q}{K} = \frac{q}{k}$$

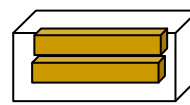


Examples of Q=RK Problems

- Travel: $D=RT$
Distance = Rate of distance/time • Time
- Mileage: $D=MV$
Distance = Mileage rate of miles/gallon • Gallons
- Cost: $C=PK$
Cost = Price per /unit • Kwantity of units
- Work: $W=RT$
Work = Rate of work/time • Time
- Coin: $T=VC$
Total value = Value per coin • Coin kwantity



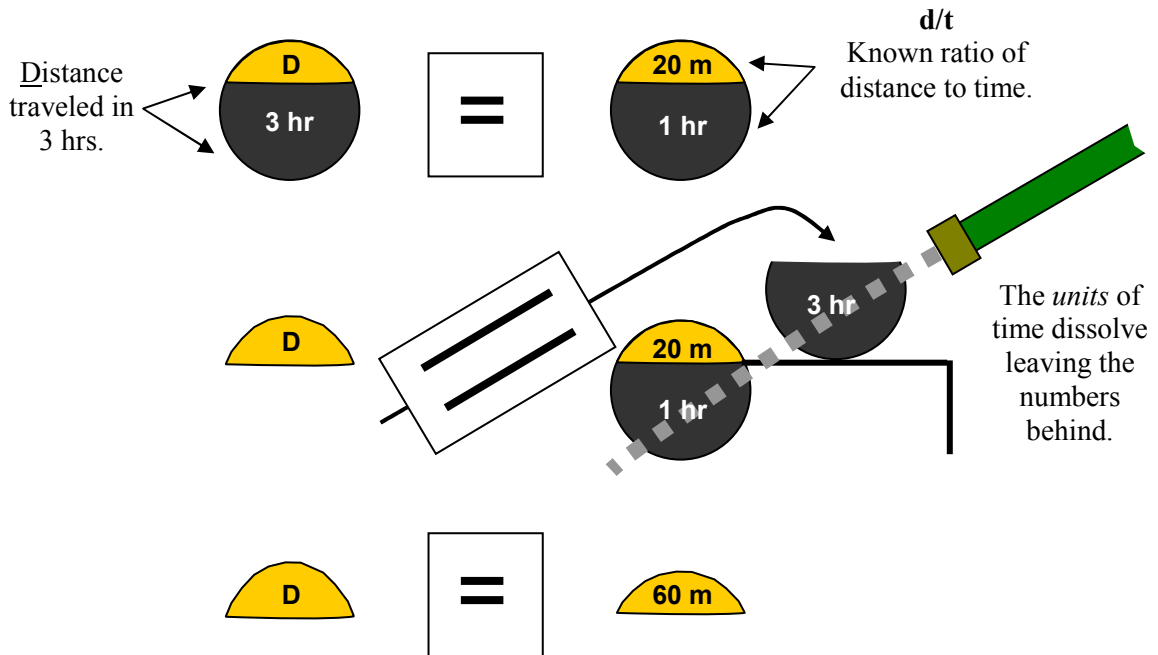
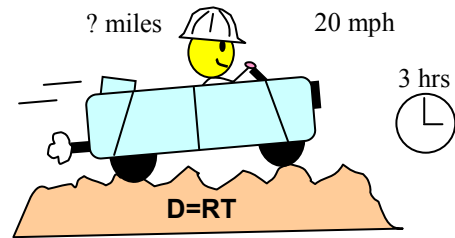
BrainAid
Imagine a car traveling on a dirt (D=RT) road.



BrainAid
Imagine seeing a pack of items (C=PK see-pack) you want to buy.

**Q-Rock Rick drives 20 miles per hour.
How far does he travel in 3 hours?**

Unknown Gold: Distance (in 3 hours)



How long will it take him to travel 80 miles?

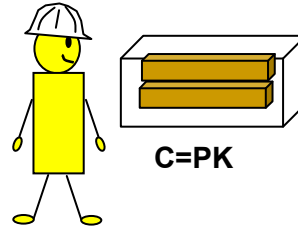
Unknown Gold: Time (to travel 80 miles)

How fast must he drive to travel 120 miles in 5 hours?

Unknown Gold: Rate (distance in 1 hour)

Q-Rock Rick wants to buy wood beams that cost \$3 for 2. How much would 6 beams cost?

Unknown Gold = Cost (of 6 beams)



How many beams can Rick buy for \$6?

Unknown Gold = Kquantity of Beams (for \$6)

What is the price per beam if 25 beams sell for \$30?

Unknown Gold = Price (rate per 1 beam)