Positional Notation

Digits represent different values depending on their position or "place" in a number.

Digit
Numerical symbol
Number
One or more digits

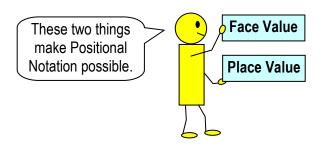
The magic of Positional Notation is that any number, large or small, can be represented by these ten digits and a decimal point:

0 1 2 3 4 5 6 7 8 9 .

Compare the compactness of Positional Notation (based on Hindu-Arabic numbering, which we use) to non-positional numbering systems.

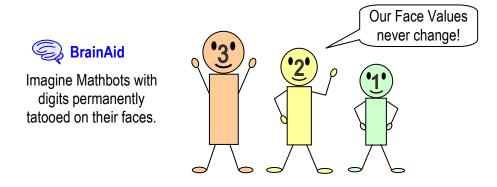
With non-positional systems, the symbols needed grow quickly as numbers get larger, making them difficult to work with.

Positional	Tick Mark	Ancient Egyptian
Notation	System	System
1		I
10	####)
100	#######################################	0
111		© ∩
222	######################################	@ ∩ ∩ I
333	######################################	@@_nn111



Face Value

The value of a digit itself, which never changes.



Place Value

The Face Value of a digit multiplied by its position in a number.



Place Value Sidewalk

Imagine that each square has a Place Value.

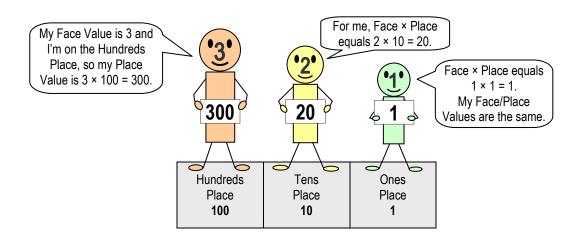
100	Hundreds Place 100	Tens Place 10	Ones Place 1	•	Tenths Place .1	Hundredths Place .01
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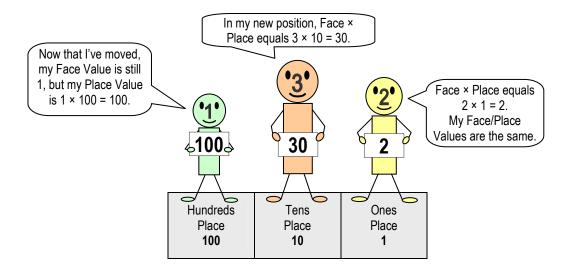
Decimal Point



Imagine Mathbots with permanent Face Values moving around on the Sidewalk calculating and displaying their changing Place Values.

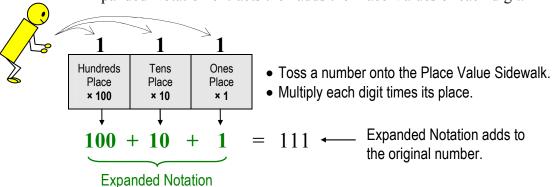
Face Value × Place = Place Value

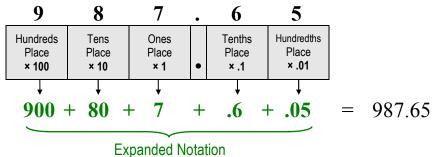




Expanded Notation

Expanded Notation extracts then adds the Place Values of each digit.





Expanded Notation

The Importance of Zero

Zero is a "place" holder for a position in a number that has no value.

Some early number systems had no concept of or symbol for zero, so a place without value would be left blank.

5		4
Hundreds	Tens	Ones
Place	Place	Place
100	10	1

For example,
"Five hundred four"
would be written as
5 4
with a gap between
the digits.
But this could be

But this could be mistaken for two separate digits or 54.

The use of zero as a placeholder eliminated the possibility of confusion.

5	0	4
Hundreds	Tens	Ones
Place	Place	Place
100	10	1



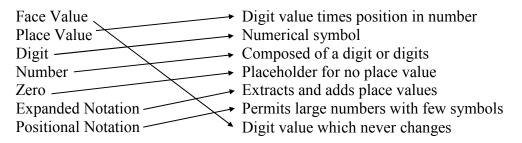
Your Turn!

Draw an arrow from each item to the best matching example.

Face Value Digit value times position in number Place Value Numerical symbol Composed of a digit or digits Digit Number Placeholder for no place value Extracts and adds place values Zero **Expanded Notation** Permits large numbers with few symbols **Positional Notation** Digit value which never changes Fill in the Place Value Sidewalk up to the Ten-Thousands Place. Ones Place 1 For each number, write the Place Value of the digit 3 in words. 3 30 300 3,000 30,000 *Rewrite the following numbers in Expanded Notation.* 145 = _____ 9,191 = ____ 54,026 = Convert the Expanded Notations back into numbers. 300 + 70 + 2 = 12,000 + 800 + 0 + 1 = 190,000 + 7,000 + 800 + 50 + 6 =

Answers follow on next page.

Answers



Ten- Thousands Place 10,000	Thousands Place 1,000	Hundreds Place 100	Tens Place 10	Ones Place 1
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3	Three
30	<u>Thirty</u>
300	Three hundred
3,000	Three thousand
30,000	Thirty thousand

$$\begin{array}{r}
145 = \underline{100 + 40 + 5} \\
9,191 = \underline{9,000 + 100 + 90 + 1} \\
54,026 = \underline{50,000 + 4,000 + 0 + 20 + 6}
\end{array}$$

Including a placeholding 0 in Expanded Notation is optional but advisable so you don't overlook a place.

$$300 + 70 + 2 = 372$$

$$2,000 + 800 + 0 + 1 = 2,801$$

$$90,000 + 7,000 + 800 + 50 + 6 = 97,856$$