## Positional Notation

Digits represent different values depending on their position or "place" in a number.

## Digit

Numerical symbol
Number
One or more digits

The magic of Positional Notation is that any number, large or small, can be represented by these ten digits and a decimal point:
0123456789 .


Compare the compactness of Positional Notation (based on Hindu-Arabic numbering, which we use) to non-positional numbering systems.

With non-positional systems, the symbols needed grow quickly as numbers get larger, making them difficult to work with.

| Positional Notation | Tick Mark System | Ancient Egyptian System |
| :---: | :---: | :---: |
| 1 | \| | I |
| 10 | H䙵\#H | $\bigcirc$ |
| 100 |  | C |
| 111 |  | C○I |
| 222 |  | cennl I |
| 333 |  | ceennnl I I |



## Face Value

The value of a digit itself, which never changes.


Imagine Mathbots with digits permanently tatooed on their faces.


## Place Value

The Face Value of a digit multiplied by its position in a number.


Place Value Sidewalk
Imagine that each square has a Place Value.

| Hundreds <br> Place <br> 100 | Tens <br> Place <br> 10 | Ones <br> Place <br> 1 | $\bullet$ | Tenths <br> Place <br> .1 | Hundredths <br> Place <br> .01 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Decimal Point |  |  |  |  |  |

Imagine Mathbots with permanent Face Values moving around on the Sidewalk calculating and displaying their changing Place Values.

## Face Value $\times$ Place $=$ Place Value



## Expanded Notation

Expanded Notation extracts then adds the Place Values of each digit.


## The Importance of Zero

Zero is a "place" holder for a position in a number that has no value.

Some early number systems had no concept of or symbol for zero, so a place without value would be left blank.


For example, "Five hundred four" would be written as 54 with a gap between the digits.

But this could be mistaken for two separate digits or 54 .

The use of zero as a placeholder eliminated the possibility of confusion.


## Your Turn!

Draw an arrow from each item to the best matching example.
Face Value Digit value times position in number

Place Value
Digit
Number
Zero
Expanded Notation Positional Notation

Numerical symbol
Composed of a digit or digits Placeholder for no place value
Extracts and adds place values
Permits large numbers with few symbols Digit value which never changes

Fill in the Place Value Sidewalk up to the Ten-Thousands Place.


For each number, write the Place Value of the digit 3 in words.

| 3 | $\square$ |
| ---: | :--- |
| 30 | $\square$ |
| 300 | $\square$ |
| 3,000 | $\square$ |
| 30,000 | $\square$ |

Rewrite the following numbers in Expanded Notation.
$\qquad$

Convert the Expanded Notations back into numbers.

$$
\begin{aligned}
300+70+2 & =\square \\
2,000+800+0+1 & =\square \\
90,000+7,000+800+50+6 & =\square
\end{aligned}
$$

Answers follow on next page.

## Answers



| Ten- | Thousands | Hundreds | Tens | Ones <br> Thousands <br> Place |
| :---: | :---: | :---: | :---: | :---: |
| Place | Place | Place | Place |  |
| 10,000 | 1,000 | 100 | 10 | 1 |


| 3 | Three |
| ---: | :--- |
| 30 | Thirty |
| 300 | Three hundred |
| 3,000 | Three thousand |
| 30,000 | Thirty thousand |



$$
\begin{aligned}
300+70+2 & = \\
2,000+800+0+1 & =92,801 \\
90,000+7,000+800+50+6 & =99,856
\end{aligned}
$$

