Problem Analysis

Key to word-problem solvability.

Equations & Unknowns

Some multiple-choice tests have an answer option similar to this:

○ Not enough information given.

Analyzing a problem can help you decide whether you have enough information *before* you attempt to solve it.

Unique Solution
To yield a single solution, problems must produce either one equation with one unknown (1EqUnk) or provide enough information to reduce more complex equations to 1EqUnks.

Multiple Solutions
Some problems may have more than one solution. For example, one equation with two unknowns (1Eq2Unk) may yield every point on the line it represents. e.g., X + Y = 5 produces (-2, 7) (-1,6) (0,5) (1,4) (2,3)…

The amount of data needed for *unique* solutions depends on the number of equations and unknowns.

Equation Type	Data for Unique Solution	Example
1EqUnk 1 Equation, 1 Unknown	None needed The single unknown may appear more than once in the equation, but it's still 1 unknown.	X + 8 = 12 To solve, get X alone. You may first need to combine multiple X's as in X + 2X = 12.
1Eq2Unk 1 Equation, 2 Unknowns	1 equivalency Equivalencies assign values to variables or relate variables to variables.	X + Y = 12 To solve for a unique X, you must be given 1 equivalency like Y=8 or Y=2X.
1Eq3Unk 1 Equation, 3 Unknowns	2 equivalencies General Rule: For 1Eqs, you need equivalencies for each Unk but one.	X + Y + Z = 12 For a unique X, you must be given 2 equivalencies like Y=6 and Z=2X.
2Eq2Unk 2 Equations, 2 Unknowns	None needed For lines that intersect. (Collinear lines share all points.) (Parallel lines share no points.)	X + Y = 12 X - Y = 6 For the unique (X,Y) where the two lines intersect, use Linear Elimination or Substitution techniques.
2Eq6Unk 2 Equations, 6 Unknowns	4 equivalencies If you're given only 3 equivalencies, the result may be solvable as a Quadratic ($ax^2+bx+c=0$) equation.	$Q_1=R_1K_1$ $Q_2=R_2K_2$ To solve for all unknowns, you need 4 equivalencies, at least 2 in each equation or with equivalencies *between* equations.

2Eq6Unk Example

Solve for $Q_1=R_1K_1$ and $Q_2=R_2K_2$

Given:

Equivalencies
- ① $Q_1=Q_2$
- ② $R_1=2$
- ③ $R_2=3$
- ④ $K_1=K_2+2$

Circled numbers link items to their given equivalencies.

$$\begin{aligned} Q_1 &\overset{①}{=} Q_2 \\ \overset{②}{R_1}K_1 &= \overset{③}{R_2}K_2 \\ 2K_1 &= 3K_2 \\ 2(K_2+2) &= 3K_2 \\ 2K_2+4 &= 3K_2 \\ -2K_2 & \quad -2K_2 \\ 4 &= K_2 \end{aligned}$$

$$\begin{aligned} K_1 &\overset{④}{=} K_2+2 \\ K_1 &= 4+2 \\ K_1 &= 6 \end{aligned}$$

$$\begin{aligned} Q_1 &= \overset{②}{R_1}K_1 \\ Q_1 &= 2(6) \\ Q_1 &= 12 \end{aligned}$$

$$\begin{aligned} Q_2 &= \overset{③}{R_2}K_2 \\ Q_2 &= 3(4) \\ Q_2 &= 12 \end{aligned}$$

"Plug-in" lines trace the flow of intermediate solutions.

Direct vs. Inverse Proportionality

Analyzing relationships between problem elements.

Directly Proportional

To maintain equivalencies, directly-proportional items must rise and fall in the *same* direction.

$$\boxed{\frac{Q}{K}} = \boxed{\frac{q}{k}}$$

As Q goes up, K goes up.

$$\frac{2\uparrow}{4\uparrow} \rightarrow \frac{4}{8}$$

(As q goes up, k goes up.)

As Q goes down, K goes down.

$$\frac{4\downarrow}{8\downarrow} \rightarrow \frac{2}{4}$$

(As q goes down, k goes down.)

Items inside D's are Directly proportional.

$$\frac{Q}{K} = \frac{q}{k}$$

As Q goes up, q goes up.

$$\frac{2\uparrow}{4} = \frac{4\uparrow}{8} \rightarrow \frac{4}{4} = \frac{8}{8}$$

(As K goes up, k goes up.)

As Q goes down, q goes down.

$$\frac{4\downarrow}{4} = \frac{8\downarrow}{8} \rightarrow \frac{2}{4} = \frac{4}{8}$$

(As K goes down, k goes down.)

Inversely Proportional

To maintain equivalencies, inversely-proportional items must rise and fall in *opposite* directions.

$$\frac{Q}{K} = \frac{q}{k}$$

As Q goes up, k goes down.

$$\frac{2\uparrow}{4} = \frac{4}{8\downarrow} \rightarrow \frac{4}{4} = \frac{4}{4}$$

(As Q goes down, k goes up.)

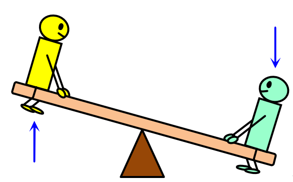
Items on the I "seesaws" are Inversely proportional.

$$\frac{Q}{K} = \frac{q}{k}$$

As K goes up, q goes down.

$$\frac{2}{4\uparrow} = \frac{4\downarrow}{8} \rightarrow \frac{2}{8} = \frac{2}{8}$$

(As K goes down, q goes up.)



Paradox!
Up in value would make *heavier* number push *down* on seesaw.

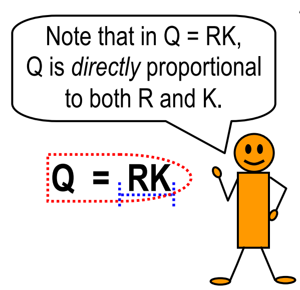
Tip: Think *opposite* direction.

$$Q = R\,K$$

As R goes up, K goes down.

$$6 = \uparrow 2 \cdot 3\downarrow$$
$$6 = 3 \cdot 2$$

(As R goes down, K goes up.)



Proportional Relationship Problem

Create an equation for: Your Chance of winning a raffle is *directly* proportional to the number of tickets you Buy and *inversely* proportional to the number of tickets Sold.

$$C = \frac{B}{S}$$

Problem Analysis:

- Directly Proportional: The *more* tickets you buy (B), the *more* chances (C) you have to win.
- Inversely Proportional: The *more* tickets sold (S), the *fewer* chances (C) you have to win.

Your Turn

Is there enough information to solve each problem? Circle "Yes" or "No."

1. Belle had 10 Fruits consisting of Apples and Oranges. Four were apples. How many were oranges?
   - Equation: $F = A + O$
   - Type: 1Eq3Unk
   - Equivalencies: $F = 10$, $A = 4$

   Enough Information? Yes / No

2. Sal drove a Distance of 30 miles at a given Rate of speed. How much Time did it take?
   - Equation: $D = RT$
   - Type: 1Eq3Unk
   - Equivalencies: $D = 30$

   Enough Information? Yes / No

3. Pia was 4 years younger than Jim. Jim was twice Pia's age? How old were they?
   - Equations: $P = J - 4 \quad J = 2P$
   - Type: 2Eq2Unk
   - Equivalencies: None

   Enough Information? Yes / No

4. Train A left the station heading west at 50 mph. At the same time Train B headed east at 60 mph. How far were they apart 3 hours later?
   - Equations: $D_A = R_A T_A \quad D_B = R_B T_B$
   - Type: 2Eq6Unk
   - Equivalencies: $R_A=50$, $R_B=60$, $T_A=3$, $T_B=3$

   Enough Information? Yes / No

Is the described relationship Directly or Inversely proportional? Circle your answer.

5. The number of trees in an orchard and the total amount of fruit. — Directly / Inversely
6. The unpopularity of a politician and his chances of being elected. — Directly / Inversely
7. The rate of speed and the distance traveled in a given amount of time. — Directly / Inversely
8. The rate of speed and the time it takes to travel a given distance. — Directly / Inversely

Create Proportional Relationship Equations

9. The Share of food that each person gets is directly proportional to the Amount of food and inversely proportional to the number of People.

10. The Quantity of gold in a rock is directly proportional to both the Percent of gold in the rock and the size of the rocK.

Answers: [1] Yes, [2] No, [3] Yes, [4] Yes, [5] D, [6] I, [7] D, [8] I, [9] S=A/P, [10] Q=PK